AS Level Physics

Chapter 5 – Mechanics

5.3.1 Linear Motion and Projectile Motion

Notes

EQUATIONS OF MOTION

SUVAT equations

There are five equations of motion that we need to learn which also include the following five quantities:

- $\boldsymbol{s} = displacement\ measured\ in\ metres\ (m)$
- $\boldsymbol{u} = initial\ velocity\ measured\ in\ metres\ per\ second\ (ms^{-1})$
- $\boldsymbol{v} = final\ velocity\ measured\ in\ metres\ per\ second\ (ms^{-1})$
- $\boldsymbol{a} = acceleration\ measured\ in\ metres\ per\ second\ squared\ (ms^{-2})$
- $\boldsymbol{t} = time\ measured\ in\ seconds\ (s)$

Sometimes the equations of motion are referred to as ***SUVAT*** equations, arising from the definitions of the kinematic quantities shown above.

EQUATION OF MOTION

The five equations of motion

The equations of motion (or SUVAT equations) can be used when **acceleration is constant** and **air resistance is negligible**. They help to describe the motion of an object.

The five equations of motion are shown below. Please note that each equation only includes four out of the five quantities shown in the opposite side:

1. $v = u + at$ ....(Doesn't include ***s***)
2. $s = \frac{v+u}{2} \times t$ ....(Doesn't include ***a***)
3. $s = ut + \frac{1}{2}at^2$ ....(Doesn't include ***v***)
4. $s = vt - \frac{1}{2}at^2$ ....(Doesn't include ***u***)
5. $v^2 = u^2 + 2as$ ....(Doesn't include ***t***)

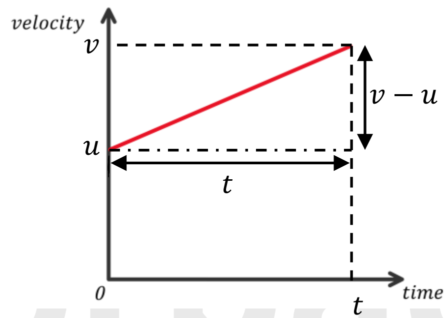
THE DERIVATION OF EQUATIONS OF MOTION

Equation 1: $\boldsymbol{v = u + at}$

To derive the first equation of motion you use a velocity time graph with a constant acceleration.

The gradient of the velocity-time graph is equal to the acceleration. Hence:

$$gradient = \frac{y_2 - y_1}{x_2 - x_1} = \frac{v - u}{t - 0}$$



As gradient is equal to acceleration we get:

$$a = \frac{v - u}{t}$$

Then you rearrange to give:

$$at = v - u$$

Therefore:

$$v = u + at$$

$$\boldsymbol{v = u + at}$$

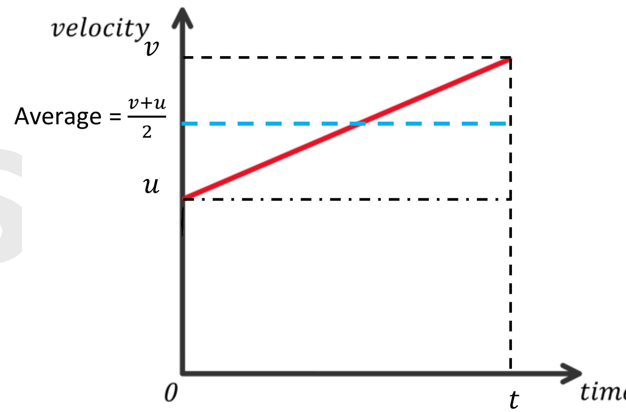
THE DERIVATION OF EQUATIONS OF MOTION

Equation 2: $\boldsymbol{s = \frac{v+u}{2} \times t}$

You can calculate the average velocity of an object by calculating the average of v and u as shown below:

$$\frac{v + u}{2}$$

You can see the average velocity represented by the blue dashed line.



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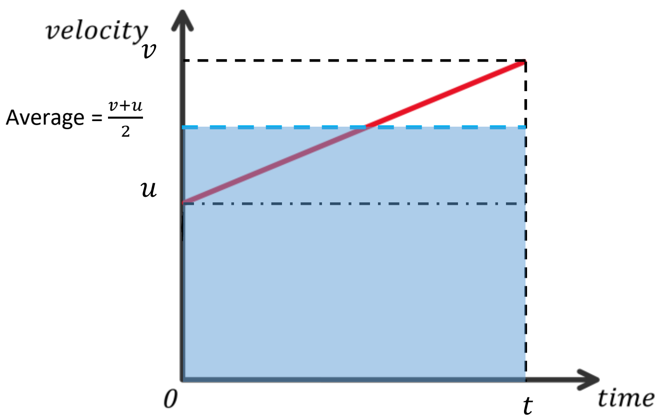
THE DERIVATION OF EQUATIONS OF MOTION

Equation 2: $s = \frac{v+u}{2} \times t$

Now we can calculate the distance travelled by the object by calculating the area under the blue dashed line (blue rectangular area).



To calculate the area you will need the below formula:

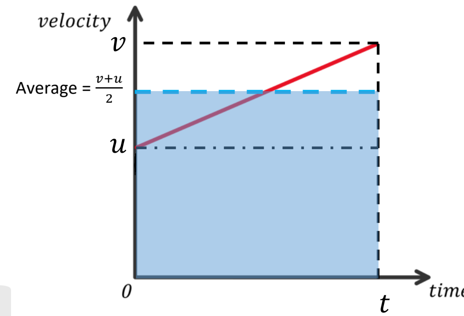
$$distance\ travelled = average\ velocity \times time$$

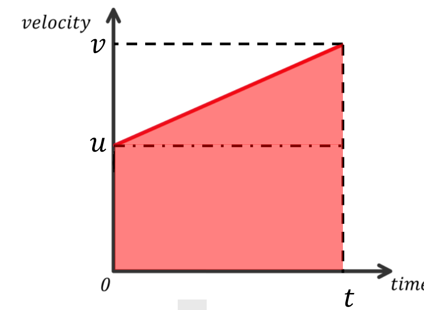
$$s = \frac{v+u}{2} \times t$$

THE DERIVATION OF EQUATIONS OF MOTION

Equation 2: $s = \frac{v+u}{2} \times t$

The blue rectangular area under the blue dashed line is the same as the red area under the sloping line.





THE DERIVATION OF EQUATIONS OF MOTION

Equation 3: $s = ut + \frac{1}{2}at^2$

To derive this equation we need equations **1** and **2**:

$\boldsymbol{v = u + at}$..**1**

$\boldsymbol{s = \frac{v+u}{2} \times t}$ **2**

Now substitute $v = u + at$ into $s = \frac{v+u}{2} \times t$ to give:

$$s = \left(\frac{u + at + u}{2}\right) \times t = \left(\frac{2u + at}{2}\right) \times t = \left(\frac{2u}{2} + \frac{at}{2}\right) \times t$$

$$s = \left(u + \frac{1}{2}at\right) \times t$$

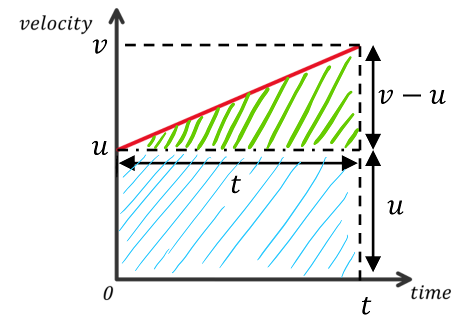
simplify to give:

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

THE DERIVATION OF EQUATIONS OF MOTION

Equation 3: $s = ut + \frac{1}{2}at^2$ **alternative method**

This is the same graph we used in the previous page and now we will use this graph to calculate the displacement.



We know the displacement = area under the velocity time graph.

Now the area under the graph can be split into a **triangle** and a **rectangle** and the sum of those two areas will give us the displacement.

$$Area\ of\ the\ rectangle = width \times height = ut$$

$$Area\ of\ the\ trianlge = \frac{1}{2} \times base \times height = \frac{1}{2} \times t \times (v - u)$$

$$\boldsymbol{We\ know\ from\ equation\ 1\ v = u + at\ therefore\ v - u = at\ so:}$$

$$Area\ of\ a\ triangle = \frac{1}{2} \times t \times at = \frac{1}{2}at^2$$

To get the total displacement we add the area of the triangle and the rectangle together to give:

$$s = area\ of\ the\ rectangle + area\ of\ the\ triangle$$
$$s = ut + \frac{1}{2}at^2$$

THE DERIVATION OF EQUATIONS OF MOTION

Equation 4: $s = vt - \frac{1}{2}at^2$

To derive this equation we need equations 1 and 2:

$v = u + at$……………………………………………………………………………1

$s = \frac{v+u}{2} \times t$ ………………………………………………………………………2

From rearranging equation 1 *we get* $u = v - at$

Now substitute $u = v - at$ into $s = \frac{v+u}{2} \times t$ to give:

$$s = \left(\frac{v + (v - at)}{2}\right) \times t = \left(\frac{2v - at}{2}\right) \times t = \left(\frac{2v}{2} - \frac{at}{2}\right) \times t$$

$$s = \left(v - \frac{1}{2}at\right) \times t$$

simplify to give:

$$s = vt - \frac{1}{2}at^2$$

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THE DERIVATION OF EQUATIONS OF MOTION

Equation 5: $v^2 = u^2 + 2as$

Using equation 1: $v = u + at$, we rearrange to get $t = \frac{(v-u)}{a}$

Substituting t *into equation* 2: $s = \frac{(u+v)}{2} \times t$ *gives us:*

$$s = \frac{(u+v)}{2} \times \frac{(v-u)}{a} = \frac{(u+v)(v-u)}{2a} = \frac{(uv - u^2 + v^2 - uv)}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$v^2 - u^2 = 2as$$

This gives us:

$$v^2 = u^2 + 2as$$

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EXAMPLE QUESTIONS

Example 1:

A car accelerates from rest to a velocity of 12 m/s in a time of 5 seconds. What is its acceleration?

$$s = not\ given$$
$$u = 0ms^{-1}$$
$$v = 12\ ms^{-1}$$
$$a = ?\ \text{(what we need to find out)}$$
$$t = 5\ s$$

In the question it tells us that a car accelerates from rest that means the initial velocity (u) is zero.

Note: only includes four out of the five quantities.

Make a note of all the values you have. This makes it easier to pick the correct equation and just substitute the values into the equation.

We use equation 1 because it doesn't include s

$$v = u + at$$
$$12\ ms^{-1} = 0\ ms^{-1} + (a)(5\ s)$$
$$12 = 5a$$
$$a = \frac{12}{5} = 2.4\ ms^{-2}$$

Therefore the acceleration of the car is $2.4\ ms^{-2}$.

EXAMPLE QUESTIONS

Example 2:

A horse accelerates at $0.4\ ms^{-2}$ for 15 seconds to reach a final velocity of $14\ ms^{-1}$. What is the initial velocity?

$$s = not\ given$$
$$u = ?\ (what\ we\ have\ to\ find\ out)$$
$$v = 14\ ms^{-1}$$
$$a = 0.4\ ms^{-2}$$
$$t = 15\ s$$

Make a note of all the values you have. This makes it easier to pick the correct equation and just substitute the values into the equation.

We use equation 1 because it doesn't include s

$$v = u + at$$
$$14\ ms^{-1} = u + (0.4\ ms^{-2})(15\ s)$$
$$14 = u + 6$$
$$u = 14 - 6 = 8\ ms^{-1}$$

Therefore the initial velocity is $8\ ms^{-1}$.

EXAMPLE QUESTIONS

Example 3:

An object travelling at 13 ms^{-1} decelerates at 4 ms^{-2} for 3s. Calculate its displacement.

$$s = ?$$
$$u = 13\ ms^{-1}$$
$$v = not\ given$$
$$a = -4\ ms^{-2}$$
$$t = 3\ s$$

Negative 3 because the object is decelerating.

Make a note of all the values you have. This makes it easier to pick the correct equation and just substitute the values into the equation.

We use equation 3 because it doesn't include v

$$s = ut + \frac{1}{2}at^2$$
$$s = (13\ ms^{-1})(3\ s) + \frac{1}{2}(-4\ ms^{-2})(3\ s)^2$$
$$s = 39 - 18$$
$$s = 21\ m$$

Therefore the displacement of the object is 21 m.

EXAMPLE QUESTIONS

Example 4:

A car accelerates from rest for 6 seconds, and has a displacement of 55m. What is its acceleration?

Note u is zero because the car accelerates from rest, therefore the initial velocity is zero.

$$s = 55\ m$$
$$u = 0\ ms^{-1}$$
$$v = not\ given$$
$$a = ?$$
$$t = 6s$$

Make a note of all the values you have. This makes it easier to pick the correct equation and just substitute the values into the equation.

We use equation 3 because it doesn't include v

$$s = ut + \frac{1}{2}at^2$$
$$55\ m = (0\ ms^{-1})(6\ s) + \frac{1}{2}(a)(6\ s)^2$$
$$55 = 18a$$
$$a = \frac{55}{18} = 3.06\ ms^{-2}\ (2\ d.p.)$$

Therefore the acceleration of the car is $3.06\ ms^{-2}$.

EXAMPLE QUESTIONS

Example 5:

A bike accelerates at $2\ ms^{-2}$ from an initial velocity of $4\ ms^{-1}$, and has a displacement of 12 m. Calculate the velocity of the car.

$$s = 12\ m$$
$$u = 4\ ms^{-1}$$
$$v = ?$$
$$a = 2\ ms^{-2}$$
$$t = not\ given$$

Make a note of all the values you have. This makes it easier to pick the correct equation and just substitute the values into the equation.

We use equation 5 because it doesn't include t

$$v^2 = u^2 + 2as$$
$$v^2 = (4\ ms^{-1})^2 + 2(2\ ms^{-2})(12\ m)$$
$$v^2 = 16 + 48$$
$$v^2 = 64$$
$$v = \sqrt{64} = 8\ ms^{-1}$$

Therefore the velocity of the car is $8\ ms^{-1}$.

EXAMPLE QUESTIONS

Example 6:

A boat accelerates from rest at $4\ ms^{-2}$ reaching a final velocity of $15\ ms^{-1}$. Calculate the displacement of the motorbike.

$$s = ?$$
$$u = 0\ ms^{-1}$$
$$v = 15\ ms^{-1}$$
$$a = 4\ ms^{-2}$$
$$t = not\ given$$

Note u is zero because the car accelerates from rest, therefore the initial velocity is zero.

Make a note of all the values you have. This makes it easier to pick the correct equation and just substitute the values into the equation.

We use equation 5 because it doesn't include t

$$v^2 = u^2 + 2as$$
$$(15\ ms^{-1})^2 = (0\ ms^{-1})^2 + 2(4\ ms^{-2})(s)$$
$$225 = 8s$$
$$s = 28.1\ m\ (1\ d.p.)$$

Therefore the displacement of the car is 28.1 m.

ACCELERATION OF FREE FALL (g)

Free fall, g = $9.81\ ms^{-2}$

- The Earths gravitational pull causes objects to accelerate as they fall.
- Free fall describes this acceleration acting on an object as $\boldsymbol{g}$.
- For objects on the surface of the Earth, $\boldsymbol{g}$ is the acceleration due to gravity acting on an object and unless you are given another value the magnitude of $g = 9.81\ ms^{-2}$.
- This value varies across the Earths surface.
- The value is greatest nearer to the poles and decreases nearer the equator as well as at higher altitudes.
- If an object is in free fall, this mean that the only force acting on an object is its weight due to gravity.
- For an object falling vertically under the effect of gravity (and no other force), you can use the equations of motion (from the previous pages) because the object has constant acceleration.
- For you to use the equations of motion you just need to change acceleration from a to g in the equations: e.g.

1. $v = u + \boldsymbol{g}t$
2. $s = \frac{v+u}{2} \times t$
3. $s = ut + \frac{1}{2}\boldsymbol{g}t^2$
4. $s = vt - \frac{1}{2}\boldsymbol{g}t^2$
5. $v^2 = u^2 + 2\boldsymbol{g}s$

- Acceleration is a vector quantity and so direction matters.
- g always acts vertically downwards.

ACCELERATION OF FREE FALL (g)

Example:

A person drops a ball from a height of $15\ m$. What velocity does the ball hit the ground at?

First choose downwards direction as positive

s= 15 m, u = $0\ ms^{-1}$, v = ? ms^{-1}, a = g = $+9.81\ ms^{-2}$, t = ?

Positive 9.81 because acceleration due to gravity acts downwards.

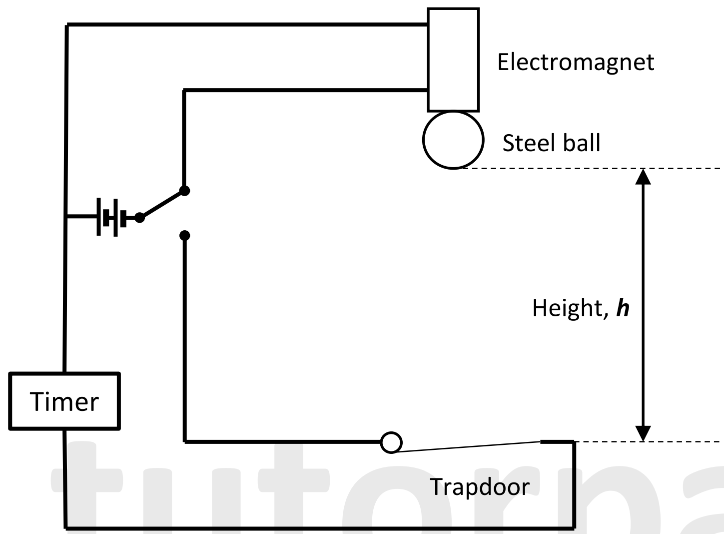
We need to find v so using:

$$v^2 = u^2 + 2as$$

We get:

$$v^2 = (0\ ms^{-1})^2 + 2(9.81\ ms^{-2})(15\ m)$$
$$v^2 = 294.3$$
$$v = \sqrt{294.3}$$
$$v = 17.2\ ms^{-1}$$

EXPERIMENT TO MEASURE ACCELERATION DUE TO GRAVITY



This set up helps you to determine the value of acceleration due to gravity ($g = 9.81\ ms^{-2}$).

Method:

1) The steel ball is held by the electromagnet above the trap door.

2) Measure the height, **h,** from the bottom of the ball to the trapdoor.

3) Switch the electromagnet off and this will release the ball and start the timer simultaneously.

4) The ball falls through the trapdoor, breaks the circuit and stops the timer.

EXPERIMENT TO MEASURE ACCELERATION DUE TO GRAVITY

Theory

From the measured height, **h,** and the time taken for the ball to break the trapdoor given by the timer, the acceleration of gravity can be calculated using:

$$h = ut + \frac{1}{2}gt^2$$

$$we\ know\ the\ initial\ velocity, u = 0\ ms^{-1}$$

$$Therefore, h = \frac{1}{2}g\ t^2$$

$$compare\ to: \quad y = mx + c$$

So, plotting a graph of h against t^2 gives a line of best fit, through the origin, having a **gradient = ½ g,** therefore.

$$acceleration\ of\ free\ fall, \mathbf{g} = 2 \times gradient\ of\ {}^{h}/_{t^2}\ graph$$

Sources of error:

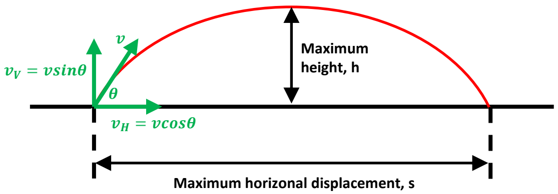
1) If the electromagnet's current is too large there will be a delay in releasing the ball after the current is switched off and the clock is triggered. Adjust the current in the electromagnet so that it just about supports the ball.

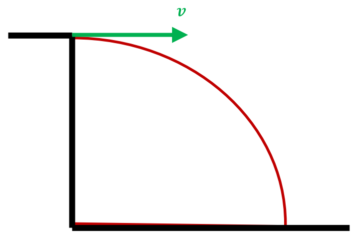
2) The measurement of h. Using a ruler, you'll have an uncertainty of about 1 mm.

Points to note:

This experiment assumes that air resistance is negligible and the magnetism of the electromagnet decays instantly.

PROJECTILE MOTION





If an object is thrown in any direction it will follow a curved (or parabolic) path through the air. Acceleration due to gravity only acts on the object vertically downwards and the object has no horizontal acceleration.

The graph above shows the path of an object thrown at an angle above the ground freely moving under the influence of gravity. This motion is known as a projectile motion.

- Horizontal motion has constant velocity (therefore zero acceleration).
- Vertical motion has uniform acceleration (acceleration of free fall, g).

Method to solve projectile motion questions:

1) Resolve the initial velocity into its horizontal and vertical components.
2) Use the vertical component (v_V) to work out how long the object is in the air for and/or the maximum height the object can reach (h).
3) Use the horizontal component (v_H) to work out the maximum horizontal displacement (s) of the object.
4) Remember when doing these questions **air resistance is negligible** and **acceleration due to gravity is constant** $(g = 9.81\ ms^{-2})$ and always **acts downwards**.

PROJECTILE MOTION SUMMARY

Resolve the vector:

Resolve the initial velocity into its horizontal and vertical component:

$$\textbf{Horizontal component}, v_H = vcos\theta$$
$$\textbf{Vertical component}, v_V = vsin\theta$$

Horizontal direction:

The horizontal component remains the same (constant) throughout its motion and gravity only acts vertically downwards therefore there is no horizontal acceleration caused by gravity that can affect the motion of the object. Therefore:

Horizontal displacement = horizontal velocity x flight time

$$s = v_H \times t$$

It's like using distance = speed x time

Vertical direction:

The acceleration is constant, equal to $g = 9.81\ ms^{-2}$.

The equations of motion apply:

$$v = u + gt\ ; h = \frac{1}{2}(u+v)t$$
$$h = ut + \frac{1}{2}gt^2\ ; v^2 = u^2 + 2gh$$

At maximum height, vertical velocity = 0

Please see the **'5.3.2 Linear Motion and Projectile Motion Worked Examples'** pack for **exam style questions.**

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