AS Level Physics

Chapter 9 – Electrical Circuits

9.1.1 Series and Parallel Circuit

Notes

Kirchhoff's Laws

Kirchhoff's First Law

The total current leaving, any junction in a circuit, is equal to the total current entering the junction.

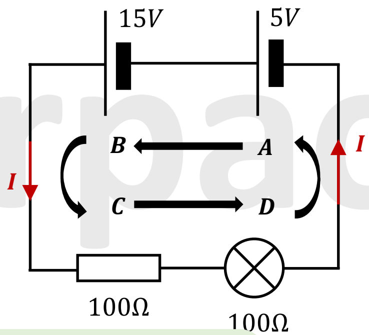
$$i.e. \sum I_{in} = \sum I_{out}$$

Kirchhoff's Second Law

Consider the opposite circuit:

Trace the movement of 1 coulomb of charge around the circuit.

Electrical energy is delivered to each coulomb as the charge passes through the first cell with $5\,V$ and then through the second cell with $15\,V$. The charge then flows through the $100\,\Omega$ resistor and a filament bulb. In each of those components, the electrical energy is converted to heat in the resistor and heat and light in the bulb.



At A: 5J gained by the charge.

At B: 15J gained by the charge.

Total energy gained = 20J

At C: 10J lost by the charge

At D: 10J lost by the charge

Total energy lost = 20J

$W = QV$
$W = 1C \times 5V$
At $A = 5J$

At $B = 1C \times 15V = 15J$

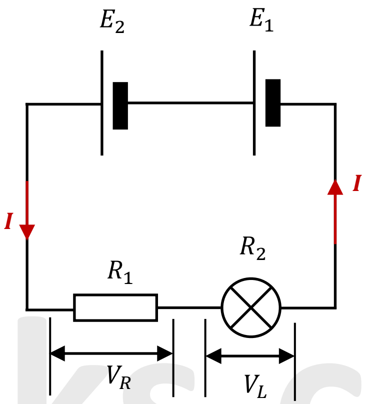
Calculate current first:
I=V/R=20/200=0.1A
At C: V=IR=0.1 x 100 = 10V
$W = QV$
$W = 1C \times 10V$
At $A = 10J$

At D: V=IR=0.1 x 100 = 10V
W = UV = $1C \times 10V = 10J$

Kirchhoff's Laws

Kirchhoff's Second Law

So the charge has lost as much energy as it gained by the time it completes the circuit. This is an example of conservation of energy. So we can conclude:



Energy supplied per coulomb by the cells (i.e. the e.m.f., E) = The sum of the energies converted per coulomb in each component (i.e. the sum of the p.d.'s)

$$\therefore\ E_1 + E_2 = V_R + V_L$$

And since the current (I) is constant throughout a SERIES circuit:

$$\boldsymbol{E_1 + E_2 = IR_1 + IR_2}$$

Kirchhoff's Second Law, as expressed in the above equation, states:

The sum of e.m.f.'s, in any closed loop in a circuit, is equal to the sum of the p.d.'s around the loop

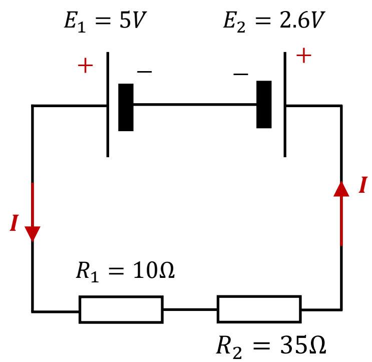
$$\boldsymbol{Net\ e.m.f. = sum\ of\ the\ p.d.'s}$$

$$\sum E = \sum IR$$

Kirchhoff's Laws Examples

Worked example 1:

Determine the current (I) in the circuit shown opposite.



Use Kirchhoff's second law to determine the current (I) in the circuit:

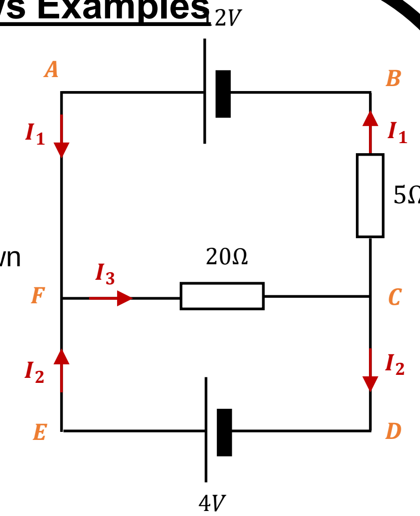
$$Net\ e.m.f = sum\ of\ the\ p.d.'s$$
$$E_1 - E_2 = IR_1 + IR_2$$
$$5 - 2.6 = (I \times 10) + (I \times 35)$$
$$2.4 = 45I$$
$$I = \frac{2.4}{45} = 0.053A$$

Current in the circuit is 0.05A.

Kirchhoff's Laws Examples

Worked example 2:

Use Kirchoff's Laws to calculate the currents I_1, I_2 and I_3 in the circuit shown opposite.



Label the closed loops **ABCDEF** as shown.

Applying Kirchhoff first law to point F:

$$I_3 = I_1 + I_2 \quad \dots\dots\dots\dots\dots\dots\dots.(1)$$

Applying Kirchhoff second law to loop FCDEF:

$$4 = 20I_3 \quad so\ I_3 = \frac{4}{20} = \mathbf{0.2A}$$

Applying Kirchhoff 2nd Law to loop ABCFA:

$$12 = 20I_3 + 5I_1 = (20 \times 0.2) + 5I_1 = 4 + 5I_1$$
$$I_1 = \frac{8}{5} = \mathbf{1.6A}$$

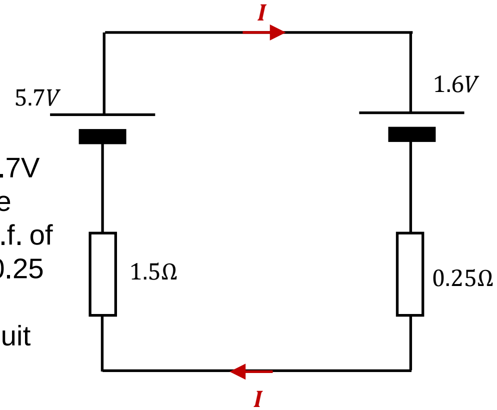
Substituting for I_1 & I_2 in equation (1)

$$0.2 = 1.6 + I_2 \quad so\ I_2 = 0.2 - 1.6 = \mathbf{-1.4A}$$

The negative sign tells us that I_2 flows in a direction opposite to that chosen.

Kirchhoff’s Laws Examples

Worked example 3:



Battery chargers with an e.m.f. of 5.7V and an internal resistance of 1.5 are used to recharge a cell with an e.m.f. of 1.6V and an internal resistance of 0.25 in the circuit diagram opposite. Determine the current (I) in the circuit shown opposite.

Use Kirchhoff’s second law to determine the current (I) in the circuit:

$$Net\ e.m.f = sum\ of\ the\ p.d.'s$$
$$E_1 - E_2 = IR_1 + IR_2$$
$$5.7 - 1.6 = (I \times 1.5) + (I \times 0.25)$$
$$4.1 = 1.75I$$
$$I = \frac{4.3}{1.75} = 2.457A$$

Current in the circuit is 2.46A.

Kirchhoff’s Laws Examples

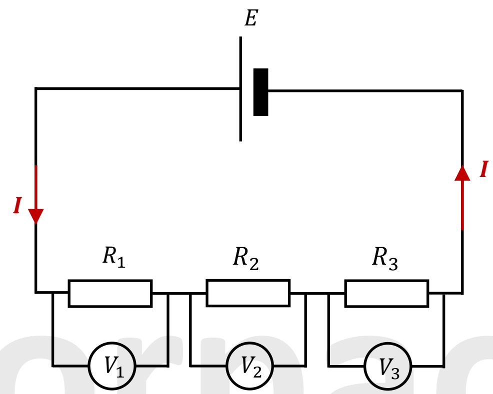
Continue to the next page.

Resistors in Series

When resistors are connected in series:

- The **current** flowing through the resistors **is the same**.
- The **p.d.** of the supply is **shared** between them.

Consider the following three resistors of resistance R_1, R_2 and R_3 connected in series as shown:



- Kirchhoff's 1st Law states:

The current (I) is constant throughout the circuit. As a result, the current in each resistor is the same.

- Kirchhoff's 2nd Law states:

The e.m.f. (E) is split between the components therefore:

$$E = V_1 + V_2 + V_3$$
$$V = IR, \text{so if } I \text{ is constant:}$$
$$IR_{total} = IR_1 + IR_2 + IR_3$$
$$\text{Cancelling the } I\text{'s gives:}$$
$$R_{total} = R_1 + R_2 + R_3$$

Therefore:

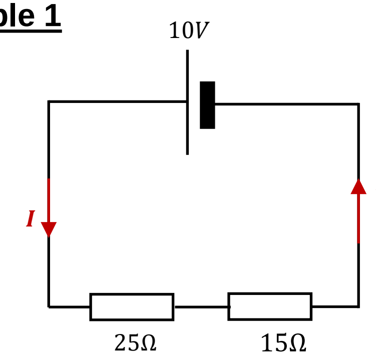
The total resistance (R_{total}), of any number of resistors, connected in series is given by:

$$\boldsymbol{R_{total} = R_1 + R_2 + R_3 \dots\dots + R_N}$$

Series and Parallel Circuit

Resistors in Series Worked Example 1

A 10V battery is connected in series with a 25 resistor and a 15 resistor. What is the potential difference (p.d.) across each resistor?



Step 1: Sketch a diagram; calculate the combined resistance:

$$\boldsymbol{R_{total} = R_1 + R_2 = 25\Omega + 15\Omega = 40\Omega}$$

Step 2: Calculate the current that flows:

$$\boldsymbol{I = \frac{V}{R} = \frac{10V}{40\Omega} = 0.25\ A}$$

Step 3: Calculate the p.d. across each resistor:

Across 20Ω: $\boldsymbol{V = IR = 0.25A \times 25\Omega = 6.25V}$

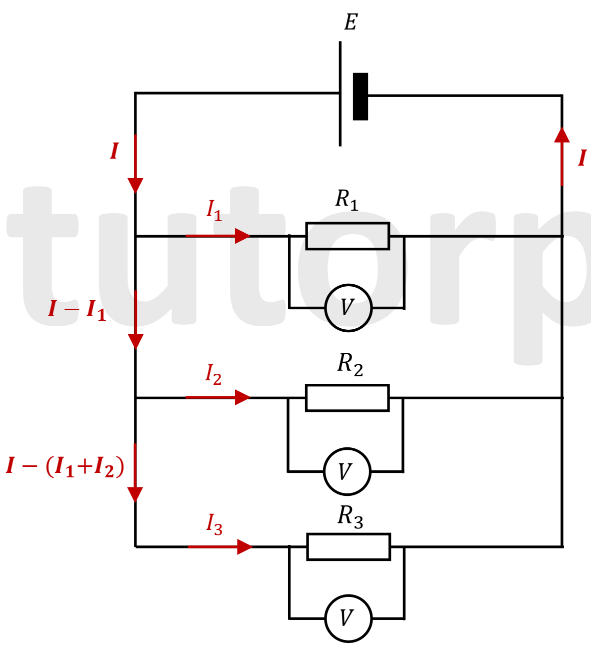
Across 5Ω: $\boldsymbol{V = IR = 0.25A \times 15\Omega = 3.75V}$

Resistors in Parallel

When resistors are connected in parallel:

- The **p.d.** across the resistors is the **same**.
- The **current** from the supply is **shared** by the resistors.

Consider the following three resistors of resistance R_1, R_2 and R_3 connected in parallel as shown:



Series and Parallel Circuit

Resistors in Parallel

Kirchhoff's First Law states:

At each junction, the current is split, so,

$$I = I_1 + I_2 + I_3 \dots\dots\dots\dots\dots \ (1)$$

Because the p.d. is the same across all of the components, each resistors p.d. is equal to V.

From the definition of resistance:

$$I = \frac{V}{R}$$

Applying this to equation (1) we get:

$$\frac{V}{R_{TOTAL}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Cancelling the V's gives:

$$\frac{1}{R_{TOTAL}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Therefore:

The total resistance (R_{total}), of any number of resistors, connected in parallel is given by:

$$\frac{1}{R_{TOTAL}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots\dots\dots + \frac{1}{R_N}$$

For resistors connected in parallel:

- The greatest current is carried by the resistor with the smallest resistance.
- The total resistance of the combination is smaller than the resistor with the smallest resistance in the combination.

Series and Parallel Circuit

Special Case for Resistors in Parallel

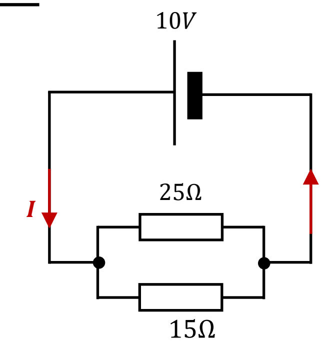
When (N) resistors with the same resistance (R) are connected in parallel, the total resistance (R_{total}) is calculated using:

$$\boldsymbol{R_{total} = \frac{R}{N}}$$

Series and Parallel Circuit

Resistors in Parallel Worked Example 1

A 25 Ω resistor and a 15 Ω resistor are connected in parallel with a 10V battery. What current flows from the battery?



Step 1: Calculate the total resistance:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{25\Omega} + \frac{1}{15\Omega} = 0.1066\ldots$$

$$\frac{1}{R_{total}} = 0.1066\ldots$$

$$R_{total} = \frac{1}{0.10666\ldots}$$

$$\boldsymbol{R_{total} = 9.375\Omega}$$

(For resistances connected in parallel, R is always less than the smallest of $\boldsymbol{R_1, R_2}$, etc...)

Step 2: Calculate the current from the combined resistance and the p.d.:

$$I = \frac{V}{R} = \frac{10\ V}{9.375\ \Omega} = 1.0666\ldots\ A$$

$$\boldsymbol{I = 1.07\ A}$$

Series and Parallel Circuit

Resistors in Series and Parallel Worked Example

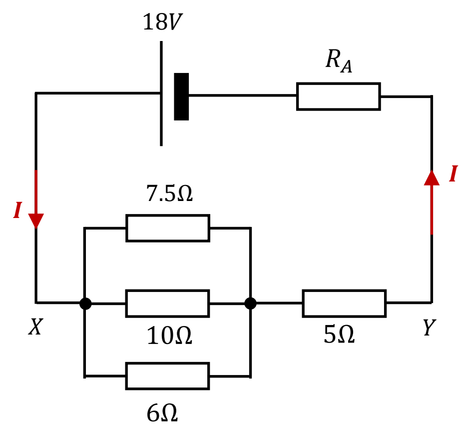
A battery of e.m.f. 18V and negligible internal resistance is connected in a circuit as shown opposite:

a) Show that the group of resistors between X and Y could be replaced with a single resistor of resistance 7.5Ω.

b) If $R_A = 25\Omega$:

i) Determine the potential difference across R_A,

ii) Calculate the current in the 7.5Ω resistor.



a) Total resistance between X and Y:

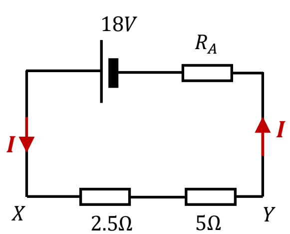
Step 1: First calculate the combined resistance of the resistors in parallel (7.5 Ω, 10 Ω and 6 Ω):

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{7.5} + \frac{1}{10} + \frac{1}{6} = \frac{2}{5}$$

$$\frac{1}{R_{total}} = \frac{2}{5}$$

$$\boldsymbol{R_{total}} = \frac{\mathbf{1}}{\left(\frac{\mathbf{2}}{\mathbf{5}}\right)} = \mathbf{2.5\ \Omega}$$

Step 2: Now that you have calculated the total resistance of the resistors in parallel you can re-draw the circuit to make it easier to visualise:



You can see the resistors are in series so you calculate the overall resistance between X and Y using:

$$R_{total} = R_1 + R_2 = 2.5\Omega + 10\Omega$$

$$\boldsymbol{R_{total} = 7.5\Omega}$$

Series and Parallel Circuit

Resistors in Parallel Worked Example 1

b) Calculate the p.d. across R_A:

To calculate the p.d. we need to use $V = IR$. We have $V = 18V$ and $R = R_A = 25\Omega$ but we need to calculate the current through R_A first. So:

Step 1: Calculate the total resistance in the circuit:

Total resistance in the circuit = 25Ω + 7.5Ω = 32.5Ω

7.5Ω is the total resistance between X and Y we calculated in the previous question.

Step 2: Calculate the current flowing through R_A:

The current through R_A can be found using:

$$\boldsymbol{I = \frac{V_{total}}{R_{total}} = \frac{18\ V}{32.5\Omega}}$$

$$\boldsymbol{I = 0.5538\ \ldots .A}$$

Always use the exact answer from your calculator until you get to the final answer and then you can round up or down to appropriate significant figures.

Step 3: Now you can use:

$$\boldsymbol{V = IR_A = (0.5538\ \ldots A)(25\Omega)}$$

Therefore p.d. across R_A is 13.8V

c) Calculate the current in the 15Ω resistor:

We know the current flowing into the group of three resistors and out of it, but not through the individual branches. But we know that their combined resistance is 5Ω (from part a) so you can work out the p.d. across the group:

$$V = IR = (0.5538 \ldots \ldots A)(2.5\Omega) = \frac{18}{13}\ V$$

The p.d. across the whole group is the same as the p.d. across each individual resistor, so you can use this to find the current through the 7.5Ω resistor:

$$I = \frac{V}{R} = \frac{\left(\frac{18}{13}\ V\right)}{7.5\Omega} = 0.1846 \ldots A$$

So the current through the 7.5Ω resistor is 0.18A.

Please see '9.1.2 Series and Parallel Circuit worked examples' pack for exam style questions.

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.