AS Level Physics

Chapter 2 – Foundations of Physics

2.3.1 Scalar and Vector Quantity

Notes

SCALAR QUANTITIES

- A scalar quantity **has *magnitude (size)*** but **NOT** a **direction**.

Examples Of Scalar Quantities

Scalar Quantities	Examples
Mass	70 kilograms (kg)
Time	1 second (s)
Distance	50 metres (m)
Speed	5 metres per second (ms^{-1})
Volume	10 cubic metres (m^3)
Temperature	273 Kelvin (K)
Density	1000 kilograms per cubic metre (kgm^{-3})
Pressure	70 Pascals (Pa)
Energy	120 Joules (J)
Potential Difference	230 Volts (V)
Frequency	50 Hertz (Hz)
Wavelength	60 metres (m)
Power	100 Watts (W)

CALCULATING SCALAR QUANTITIES

- You can add and subtract scalar quantities like normal:

Example 1:

Calculate the total mass of a 1600 kg car carrying a 75 kg man.

$$1600\ kg + 75\ kg = 1675\ kg$$

Example 2:

Water is heated from 15°C to 70°C in a kettle. Calculate the increase in temperature.

$$70\ °C - 15\ °C = 55\ °C$$

VECTOR QUANTITIES

- A vector quantity has both ***magnitude (size) and direction***.

Examples Of Vector Quantities

Vector Quantities	Examples
Displacement	15 metres (m) West
Velocity	23 metres per second (ms^{-1}) to the right
Acceleration	9.81 metres per second squared (ms^{-2}) downwards
Force	55 newtons (N) due North
Impulse	95 newton second (Ns) to the right
Momentum	550 kilogram metres per second ($kgms^{-1}$) North West
Electric Current	4 Amperes (Amps, A)
Magnetic Field	20 Tesla (T)
Electric Field	25 newtons per coulomb (NC^{-1})
Weight	750 newtons (N) downwards

Continue to the next page

DIFFERENCES BETWEEN SCALAR AND VECTOR QUANTITIES

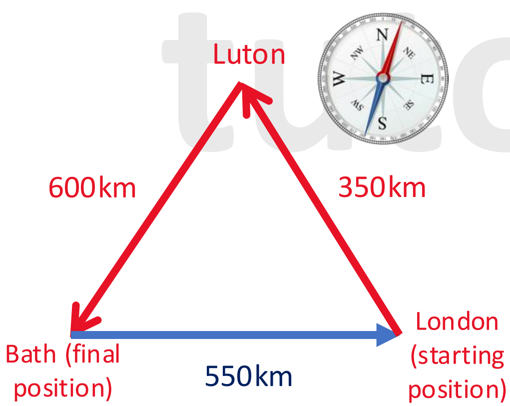
- A ***vector quantity*** has ***both magnitude (size) and direction*** whereas a ***scalar quantity only*** has a ***magnitude (size).***

Example 1: Distance and Displacement

❖ **Distance** is the complete path travelled by an object during its motion (a scalar quantity). It doesn't include a direction.

❖ **Displacement** is the distance moved in a stated direction (a vector quantity).

Lets consider a car that sets off from London and stops at Luton before going to Bath.



To determine the fuel the car needs for the journey you would need to know the total distance travelled (scalar).

If the driver just wanted to know his final position relative to the starting position, then the displacement is required (vector).

Distance travelled by the car: 600 km + 350 km = **950 km.**

Displacement: **550 km due East (90°).**

As you can see ***distance only has size***, but the ***displacement has both size and direction***.

DIFFERENCES BETWEEN SCALAR AND VECTOR QUANTITIES

This is a fancy way of saying speed is distance divided by time

Example 2: Speed VS Velocity

❖ **Speed** is the ***distance*** travelled per unit time (a scalar quantity).

❖ **Velocity** is the ***displacement*** per unit time (a vector quantity).

Lets say that the cars journey from London, to Luton, to Bath lasted 10 hours, then the speed would be:

In other words velocity is displacement divided by time

$$Speed = \frac{distance}{time} = \frac{950\ km}{10\ h} = 95\ kmh^{-1}$$

However the velocity would be:

$$Velocity = \frac{displacement}{time} = \frac{550\ km}{10\ h} = 55\ kmh^{-1} due\ East\ (90°)$$

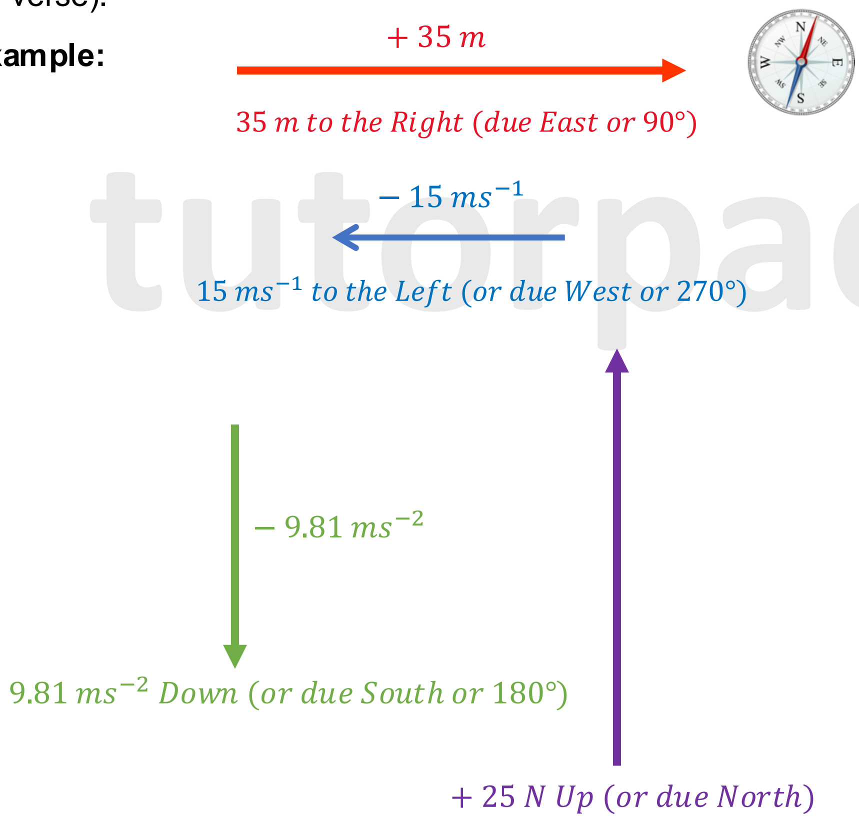
As you can see, both speed and velocity have the same units but ***speed*** only has a ***magnitude (size)*** whereas the ***velocity*** has both ***size and direction***.

VECTORS AND DIRECTION

Vectors can be represented by a written description or can be drawn as ***arrows***.

1) The ***length*** of the arrow represents the ***size*** of the vector
2) The ***direction*** of the arrow represents the ***direction*** of the vector. Positive (+) and negative (–) signs can also help indicate the direction of a vector (i.e. positive (+) can mean right or up and negative (–) can mean left or down, or vice verse).

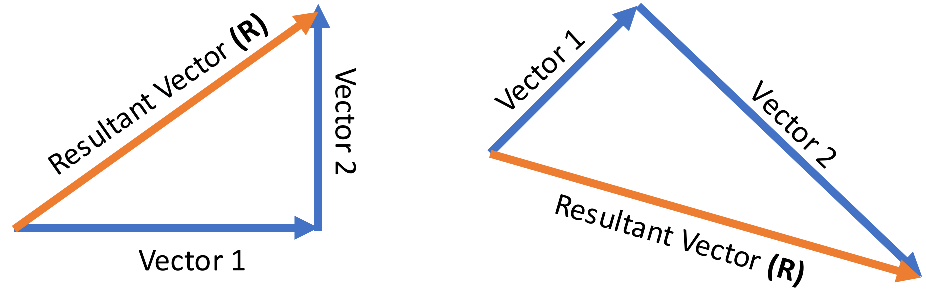
Example:



VECTOR ADDITION

The sum of two or more vectors is called the ***resultant***.

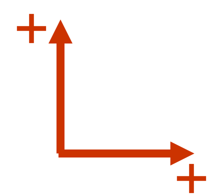
Vectors can be added using vector diagrams.



- The **resultant** of a number of forces is the single force which has the **same effect, in both magnitude and direction**, as the **sum of the individual forces**.

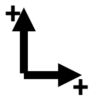
When solving vectors you need to keep in mind that the direction matters just as much as the magnitude. So you need to set up a coordinate system.

This means that if you decide vectors facing to the right are positive, then anything to the left will be negative. If you decide any vectors acting upwards are positive then anything acting downwards is negative. You can represent this coordinate system as shown below:

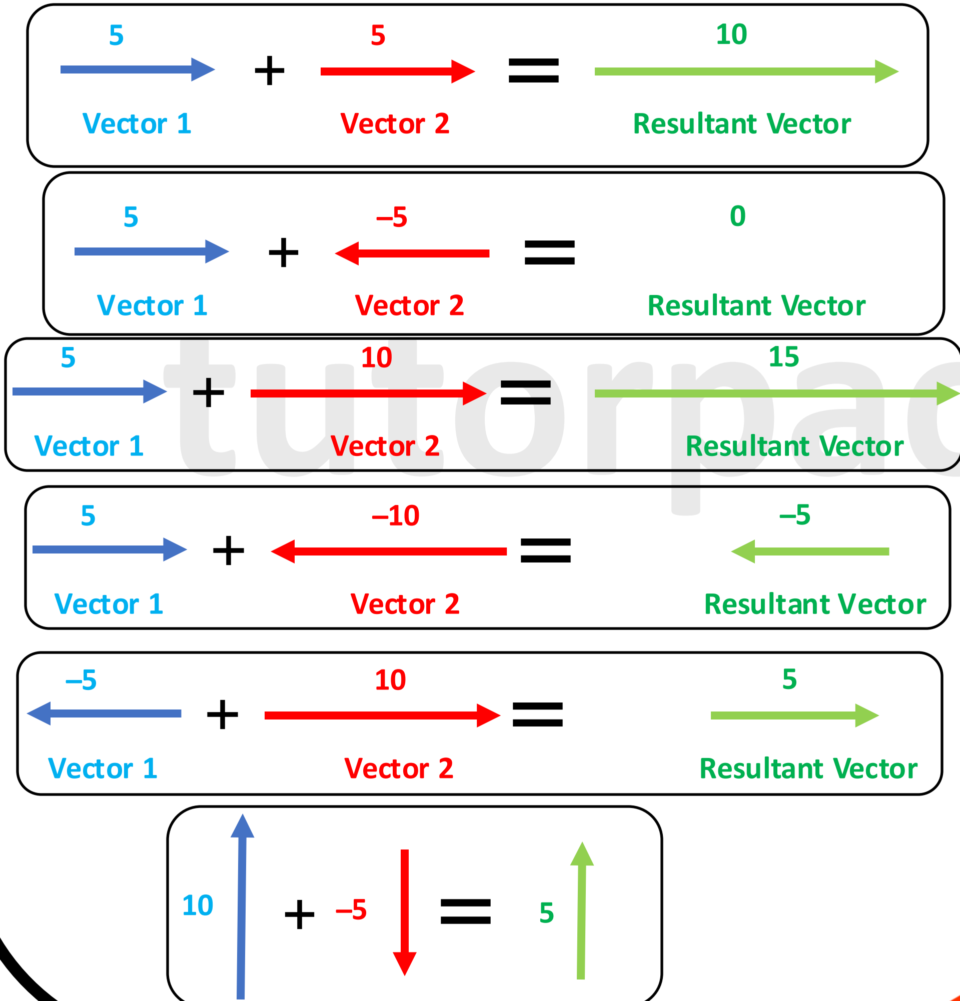


VECTOR ADDITION

Vectors Acting In The Same And Opposite Directions



This symbol is the set coordinate system I have chosen. It means vectors to the right and up are positive. Therefore, vectors facing to the left and down are negative.



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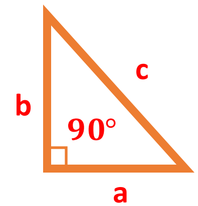
VECTOR ADDITION

Vectors acting at right angles

To figure out vectors at right angles, the Pythagoreans theorem is a useful method for determining the result (resultant).

The Pythagoreans theorem **only works** when adding two vectors that are at **right angles (90°)** to each other.

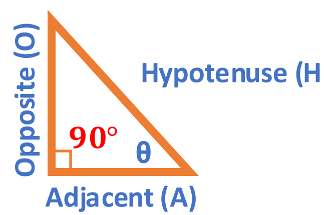


Pythagorean Theorem

$$a^2 + b^2 = c^2$$

The Pythagoreans theorem only helps you to determine the resultant (result or magnitude (size)) of a vector. However you will also need to calculate the direction of the vector too.

In order to calculate the direction of a resultant vector you will need to use trigonometric functions such as sin , cos and tan . You should remember the below from your GCSE mathematics.



$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{Opposite}{Adjacent}$$

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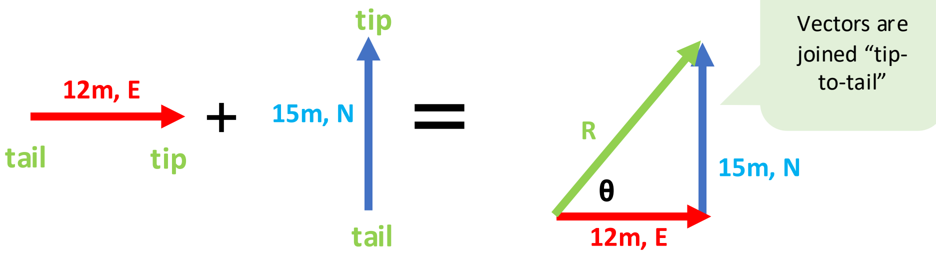
VECTOR ADDITION

Vectors Acting at right angles

Example 1:

Calculate the resultant vector of a person walking 12 metres East and then 15 metres North.



Using Pythagoreans theorem you can calculate $\boldsymbol{R}$

$$a^2 + b^2 = R^2$$

$$12^2 + 15^2 = R^2$$

$$369 = R^2$$

$$R = \sqrt{369} = \mathbf{19.2\ m\ (1\ d.p.)}$$

So the result of adding **12 m, East** plus **15 m, North** is a vector with a **magnitude of 19.2 m**. But a vector needs to have a direction as well.

To calculate the direction of the vector all you have to do is work out the angle θ using trigonometry:

$$tan\ \theta = \frac{opposite}{adjacent} = \frac{15}{12}$$

$$\theta = tan^{-1}\left(\frac{15}{12}\right) = 51.3°\ (1\ d.p.)$$

Therefore, the **Resultant (R) is 19.2 m** acting at an **angle of 51.3° above the horizontal.**

VECTOR ADDITION

Vectors acting at right angles

Example 2:

A person walks 45 m East and then 55 m South in a minute.

a) Draw a diagram showing the journey.

b) Calculate the total distance travelled.

c) Calculate the total displacement of the person.

d) Calculate the persons average speed.

e) Calculate the persons velocity.

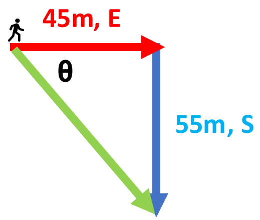
Answers on the next page.

VECTOR ADDITION

Vectors Acting at right angles

Example 2 continued:

a) Draw a diagram showing the journey:



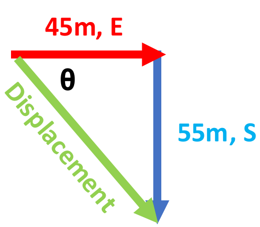
Vectors are joined "tip-to-tail"

b) Calculate the total distance travelled:

$$Total\ distance = 45m + 55m = \mathbf{100\ m}$$

c) Calculate the total displacement of the person

Remember that displacement is a vector quantity so we will have to calculate the magnitude (size) and direction of the green line.



Magnitude (size): Use Pythagoras theorem

$$a^2 + b^2 = c^2$$

$$45^2 + 55^2 = Displacement^2$$

$$Displacement = \sqrt{45^2 + 55^2} = \mathbf{71.1\ m\ (1\ d.p.)}$$

Direction: Use trigonometry

$$tan\theta = \frac{opposite}{adjacent} = \frac{55}{45}$$

$$\theta = \tan^{-1}\left(\frac{55}{45}\right) = 50.7°\ (1\ d.p.)$$

$$90° + 50.7° = \mathbf{140.7°}\ (\boldsymbol{Bearing})$$

Total displacement of the hiker is:

Displacement = 71.1 m on a bearing of 140.7°

VECTOR ADDITION

Vectors acting at right angles

Example 2 continued:

d) Calculate the persons average speed:

$$Average\ speed = \frac{distance}{time} = \frac{100\ metres}{1\ minute} = \frac{100\ metres}{60\ seconds}$$

$$= \mathbf{1.7\ m\ s^{-1}\ (1\ d.p.)}$$

e) Calculate the persons velocity

$$Velocity = \frac{displacement}{time} = \frac{71.1\ metres}{60\ seconds} = \mathbf{1.19\ m\ s^{-1}\ (2\ d.p.)}$$

But that is not the final answer. Velocity is a vector quantity so it needs to have a magnitude and a direction.

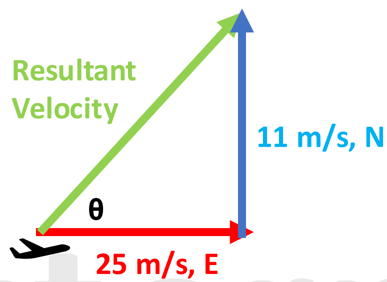
$$\boldsymbol{Velocity = 1.19\ m\ s^{-1}\ on\ a\ bearing\ of\ 140.7°.}$$

VECTOR ADDITION

Vectors Acting at right angles

Example 3:

A jet is flying with a velocity of 25 m/s due East. A crosswind is blowing with a velocity of 11 m/s due North. Calculate the resultant velocity of the plane.



Magnitude (size): Use Pythagoras theorem

$$a^2 + b^2 = c^2$$
$$25^2 + 11^2 = velocity^2$$
$$Velocity = \sqrt{25^2 + 11^2} = \mathbf{27.3\ m/s}$$

Direction: Use trigonometry

$$tan\theta = \frac{opposite}{adjacent} = \frac{11}{25}$$
$$\theta = \tan^{-1}\left(\frac{11}{25}\right) = 23.7°\ (1\ d.p.)$$
$$90° - 23.7° = \mathbf{066°}\ (\boldsymbol{Bearing})$$

Velocity = 27.3 m/s on a bearing of 066°

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VECTOR ADDITION

Vectors acting at right angles

Example 4:

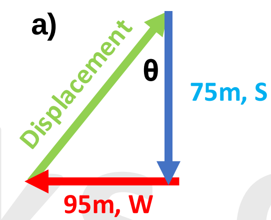
A person runs 75m due South then 95m due West.

a) Draw a diagram showing the journey .

b) Calculate the total distance travelled.

c) Calculate the total displacement of the person.

a)



b) $Total\ distance\ travelled = 75m + 95m\ = \mathbf{170}\boldsymbol{m}$

c) Total displacement:

Magnitude (size): Use Pythagoras theorem

$$a^2 + b^2 = c^2$$
$$75^2 + 95^2 = displacement^2$$
$$displacement = \sqrt{75^2 + 95^2} = \mathbf{121.0\ m\ (1\ d.p.)}$$

Direction: Use trigonometry

$$tan\theta = \frac{opposite}{adjacent} = \frac{95}{75}$$
$$\theta = \tan^{-1}\left(\frac{95}{75}\right) = 51.7°\ (1\ d.p.)$$
$$180° + 51.7° = \mathbf{231.7°}\ (\boldsymbol{Bearing})$$

Total displacement = 121 m on a bearing of 231.7°

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VECTOR ADDITION

Vectors Acting at right angles

Example 5:

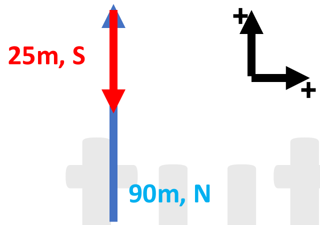
A person walks 90m due North then 25m South.

a) Draw a diagram showing the journey

b) Calculate the total distance travelled

c) Calculate the total displacement of the person

a)



b) $Total\ distance\ travelled = 90\ m + 25\ m\ = \mathbf{115\ m}$

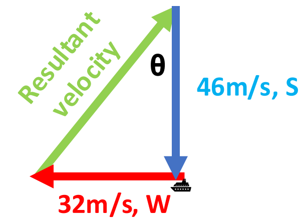
c) Total displacement = 90 m – 25m = **65 m due North**

VECTOR ADDITION

Vectors acting at right angles

Example 6:

A duck is swimming at 46 m/s due South while the wind is blowing at 32 m/s West. Calculate the resultant velocity.



Magnitude (size): Use Pythagoras theorem

$$a^2 + b^2 = c^2$$

$$32^2 + 46^2 = resultant\ velocity^2$$

$$resultant\ velocity = \sqrt{32^2 + 46^2} = \mathbf{56.0\ m/s\ (1\ d.p.)}$$

Direction: Use trigonometry

$$tan\theta = \frac{opposite}{adjacent} = \frac{32}{46}$$

$$\theta = \tan^{-1}\left(\frac{32}{46}\right) = 34.8°\ (\ 1\ d.p.)$$

$$180° + 34.8° = \mathbf{214.8°}\ (\boldsymbol{Bearing})$$

Resultant velocity = 56.0 m/s on a bearing of 214.8°

VECTOR ADDITION

Obtaining the Resultant by scale drawing

The Resultant vector can also be obtained using a scale drawing. Scale drawings help to add vectors which are not at right angles.

Example 1:

A bird travels due North for 100km. The bird changes its direction to 25° West of North and travels a further 250km. Find the displacement of the bird.

To answer this question follow the steps below:

Step 1) Choose a suitable scale (in this case say **1cm = 25km**).

Step 2) Draw a vector to represent the **100km due North** (a vertical line which is **4cm long**).

Step 3) Then draw the vector to represent **250km due 25° West of North.** To do this use a protractor to measure 25° from the **tip of the 100 km vector** and then draw a vertical line **10cm long.** Remember to join the new vectors **tail** starting at the **tip** of the **100km vector**.

Step 4) The **resultant vector** is the one which closes the triangle.

Step 5) Measure the length of the **resultant vector** using a ruler and then convert using your **scale**: 13.7cm x 25km = 342.5km

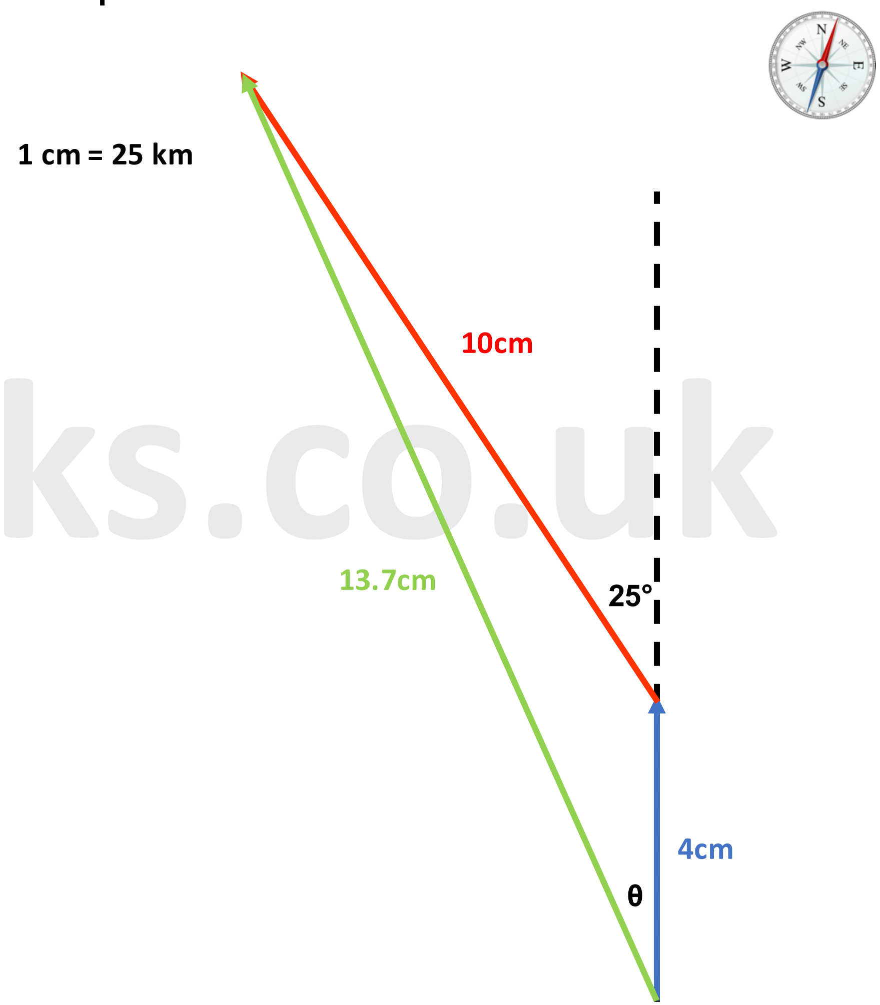
Step 6) Measure the size of the angle using a protractor. In this case it is 20°

Look at the next page to see the diagram.

VECTOR ADDITION

Obtaining the Resultant by scale drawing

Example 1 continued:



VECTOR ADDITION

Obtaining the Resultant by scale drawing

Example 2:

A ship sailing due West passes buoy X and continues to sail West for 30 minutes at a speed of 10 km/h.

The ship changes its course to 20° West of North and continues on this course for 1 hour and 30 minutes at a speed of 8 km/h until it reaches buoy Y.

a) Show that the ship sails a total distance of 17km between buoys X and Y.

b) By scale drawing, find the displacement from buoy X to buoy Y.

a)

Stage 1 - distance to X:
Distance = speed x time
Distance = 10 km/h x 0.5 hours
Distance = 5 km

Stage 2 - distance from X to Y:
Distance = speed x time
Distance = 8 km/h x 1.5 hours
Distance = 12 km

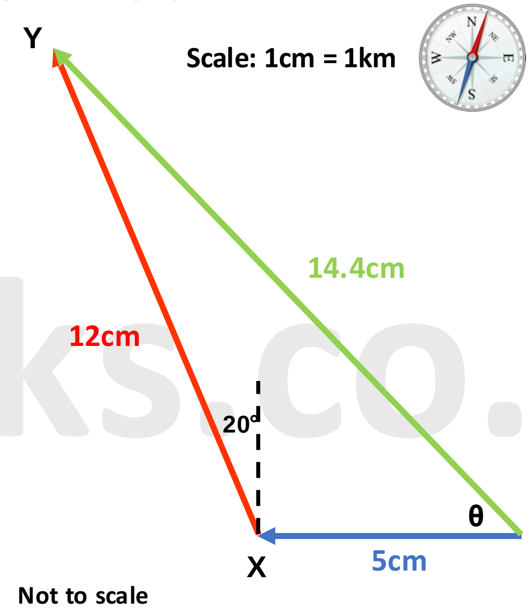
Total distance:
Distance = 5km + 12km = **17km**

VECTOR ADDITION

Vectors acting at right angles

Example 2 continued:

b) Answer this question using the steps mentioned on the previous page.



Length of Vector:
14.4cm x 1km = **14.4km**

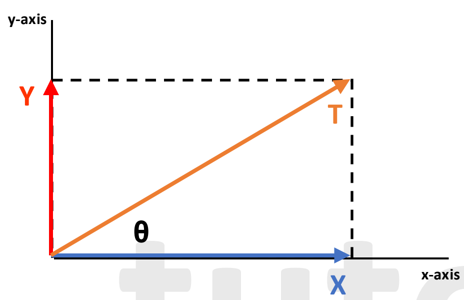
Direction of Vector:
θ = **52°**

Displacement = 14.4 km with an angle of 52°

RESOLVING VECTORS

To analyse a vector, it is essential to "break-up" or resolve a vector into it's horizontal and vertical components.

Consider a **vector T** at an angle θ to the x-axis. The **vector T** can be resolved into its **horizontal (X)** and **vertical (Y)** components using trigonometry:



$$sin\theta = \frac{opposite}{hypotenuse} = \frac{Y}{T}$$

$$So\ \boldsymbol{Y = T\ sin\theta}$$

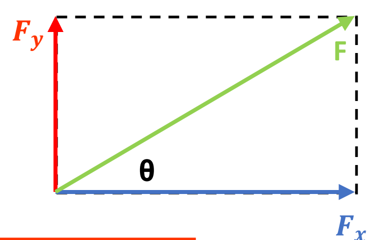
$$cos\theta = \frac{adjacent}{hypotenuse} = \frac{X}{T}$$

$$So\ \boldsymbol{X = T\ cos\theta}$$

Applying the above to any vector, the vector can be RESOLVED into a vertical and horizontal component.

The diagram opposite shows a **vector F** which has been resolved into a **vertical** $(\boldsymbol{F_y})$ and a **horizontal** $(\boldsymbol{F_x})$ **component.**

To calculate the components you use the formulas shown below:



Vertical Component: $\boldsymbol{F_y = F\ sin\theta}$

Horizontal Component: $\boldsymbol{F_x = F\ cos\theta}$

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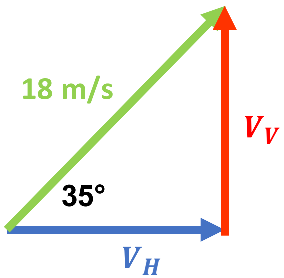
RESOLVING VECTORS

Example 1:

A ball is thrown with a velocity of 18 m/s at an angle of 35° above the ground. Calculate the horizontal and vertical components of the balls velocity.



Vertical component $(\boldsymbol{V_V})$: Solve using trigonometry

$$sin\theta = \frac{V_V}{V}$$

$$sin35 = \frac{V_V}{18}$$

$$\boldsymbol{V_V = 18\ sin35}$$

$$V_V = 10.3\ ms^{-1}\ (1\ d.p.)$$

Horizontal component $(\boldsymbol{V_H})$: Solve using trigonometry

$$cos\theta = \frac{V_H}{H}$$

$$cos35 = \frac{V_H}{18}$$

$$\boldsymbol{V_H = 18\ cos35}$$

$$V_H = 14.7\ ms^{-1}\ (1\ d.p.)$$

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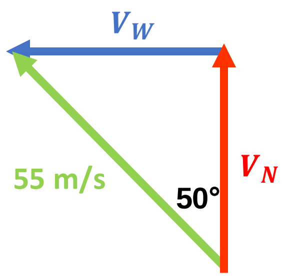
RESOLVING VECTORS

Example 2:

A duck is swimming with a velocity of 55 m/s on a bearing of 310°.
Calculate its component velocities.



North component (V_N): Solve using trigonometry

$$cos\theta = \frac{V_N}{V}$$

$$cos50 = \frac{V_V}{55}$$

$$V_N = 55 \times cos50$$

$$\mathbf{V_N = 35.4\ ms^{-1}\ (1\ d.p.)}$$

West component (V_W): Solve using trigonometry

$$sin\theta = \frac{V_W}{V}$$

$$sin50 = \frac{V_W}{55}$$

$$V_W = 55 \times sin50$$

$$\mathbf{V_W = 42.1\ ms^{-1}\ (1\ d.p.)}$$

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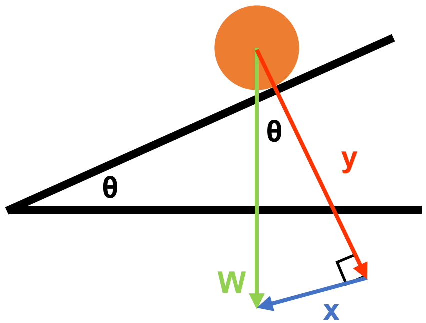
SLOPES (INCLINE PLANE) – PARALLEL AND PERPENDICULAR COMPONENTS

On a slope, the components of a vector are parallel and perpendicular to the slope.



- *W = the weight of the object*. The weight can be calculated using the formula:

$$W = mg$$

where:

W = weight measured in Newtons (N)

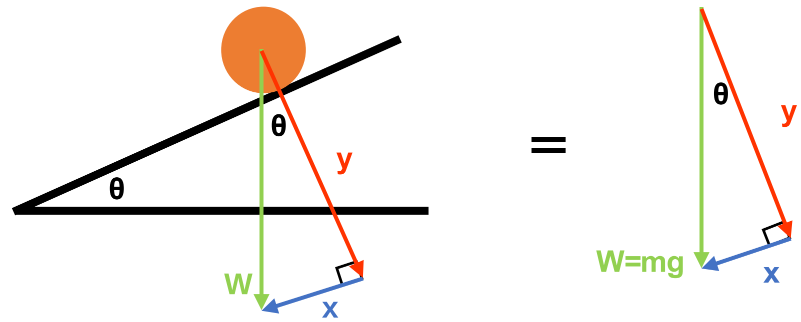
m = mass of the object measured in kilograms (kg)

g = acceleration due to gravity measured in metres per second squared ($m\ s^{-2}$)

The weight of an object always acts directly downwards from the centre point of an object. The weight component can be resolved into x and y components as shown above.

- ***x component*** = The forward force that moves the ball down the slope. This force always acts parallel to the slope.
- ***y component*** = This force balances the normal (reaction force) of the ball and so it is facing in the opposite direction and is perpendicular to the slope. This force is what keeps the ball on the slope and doesn't cause it to sink into the slope.

SLOPES (INCLINE PLANE) – PARALLEL AND PERPENDICULAR COMPONENTS



You can calculate the x and y components the same way you calculated the vertical and horizontal components of velocity in the previous examples. As you can see it's a right-angle triangle, so knowing that information you can use trigonometric functions:

Perpendicular Component (y):

$$cos\theta = \frac{adjacent}{hypotenuse} = \frac{y}{W} = \frac{y}{mg}$$

Therefore: $cos\theta = \frac{y}{mg}$

$$\text{So } \boldsymbol{y = mg\ cos\theta}$$

Parallel Component (x):

$$sin\theta = \frac{opposite}{hypotenuse} = \frac{x}{W} = \frac{x}{mg}$$

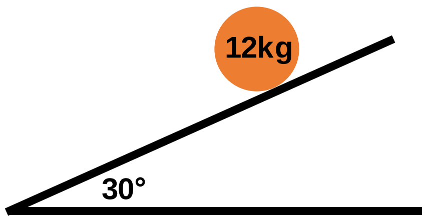
Therefore: $sin\theta = \frac{x}{mg}$

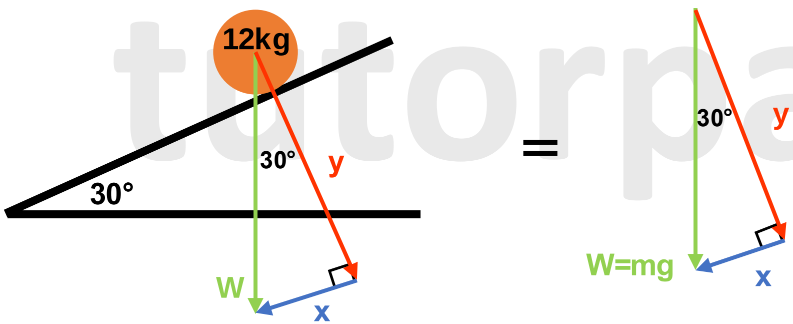
$$\text{So } \boldsymbol{x = mg\ sin\theta}$$

SLOPES (INCLINE PLANE) – PARALLEL AND PERPENDICULAR COMPONENTS

Example 1:

A 12kg mass sits on a 30° slope. Calculate the component of weight acting down (parallel) the slope.





Parallel Component (x):

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{x}{W} = \frac{x}{mg}$$

The value of g = 9.81m/s² on Earth.

$$\sin 30 = \frac{x}{(12kg)(9.81ms^{-2})} = \frac{x}{117.72\ N}$$

$$So\ \boldsymbol{x = 117.72N \times \sin 30 = 58.9N\ (1\ d.p.)}$$

Please see the **‘2.3.2 Scalar and Vector Quantity Worked Examples’** pack for exam style questions.

For more revision notes, tutorials, worked examples and more help visit
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