A2 Level Physics

Chapter 8 – Further Mechanics

8.1.1 Kinematics of Circular Motion

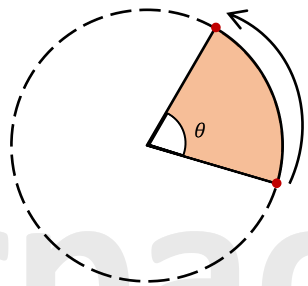
Notes

The Radian (rads)

You should be familiar with using degrees to measure angles, with a complete circle equal to $360°$.

An alternative option to using degrees is to use radians. All angle measurements in circular motion use radians, so make sure you're familiar with them before diving into this topic.

Angular Displacement

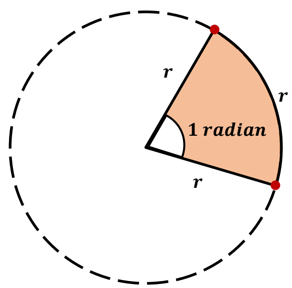


Angular displacement is the angle (θ) through which an object turns when it is moving in a circle. (Linear displacement is the equivalent quantity - when an object moves in a straight line).
Although θ can be expressed in degrees, we will use Radians (rads).

One RADIAN is the angle formed at the centre of a circle by an arc of length equal to the radius of the circle.

$$angle\ in\ radians = \frac{length\ of\ arc}{radius}$$

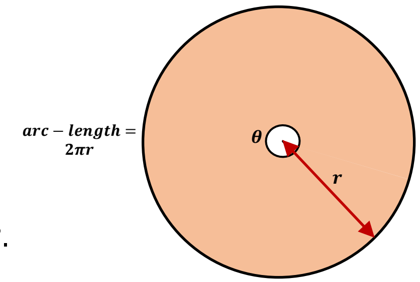
$$\theta = \frac{s}{r}$$



The Radian (rads)

For a complete circle $(360°)$, the arc-length is just the circumference of the circle $(2\pi r)$. When you divide $2\pi r$ by the radius (r) gives 2π. E.g.

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$$



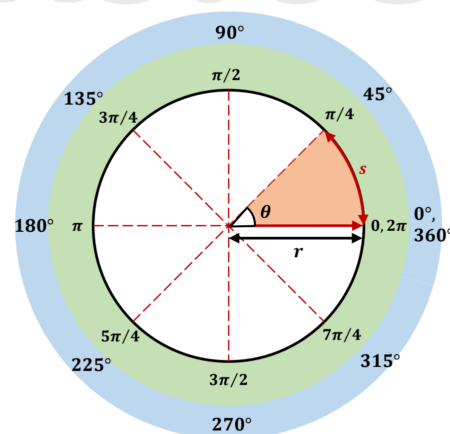
So there are 2π radians in a complete circle $(360°)$.

Therefore 1 radian is equal to about $57°$.

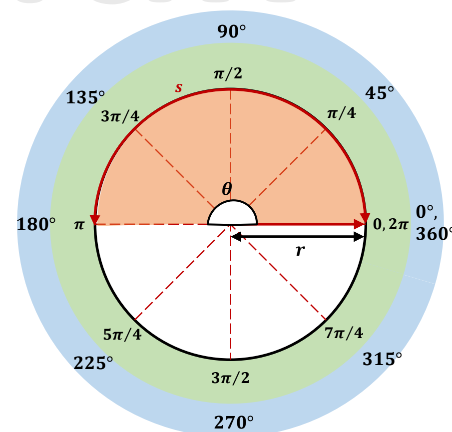
Conversation

To convert degrees to radians: multiply by $\frac{\pi}{180°}$.

To convert radians to degrees: multiply by $\frac{180°}{\pi}$.



$$\theta = \frac{\pi}{4} = 45°$$



$$\theta = \pi = 180°$$

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Angular Velocity

The angular velocity is the rate at which an object rotates. Just as linear speed, v, is defined as $distance \div time$, the angular speed, ω, is defined as $angle \div time$. The unit is rad s^{-1} (radians per second).

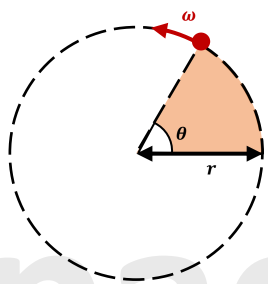
$$\omega = \frac{\theta}{t}$$

Where:

ω = angular velocity expressed as the Greek letter 'omega' (ω) measured in $rad\ s^{-1}$.

θ = angle that the object turns through in rad

t = time in s



Instantaneous Velocity

Instead of thinking about angular movement, consider the moving object's actual velocity through space (also known as the 'instantaneous velocity').

Consider an object moving in a circular path of radius, r, that moves an angle of θ rad in time, t, seconds. We know that:

$$linear\ speed = \frac{distance}{time}$$

However, because the object is moving in a circle, the distance travelled is equal to the arc length, s, that the object travels through in its circular motion, so:

$$v = \frac{s}{t}$$

But, we already know that $s = r\theta$ from the previous page, so we can plug that into the equation above to get:

$$v = \frac{r\theta}{t}$$

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Instantaneous Velocity

Rearrange to get:

$$\frac{v}{r} = \frac{\theta}{t}$$

But, because we just learned $\omega = \frac{\theta}{t}$, we can substitute it back into the equation above to get:

$$\omega = \frac{v}{r}$$

Rearranging gives us linear speed as:

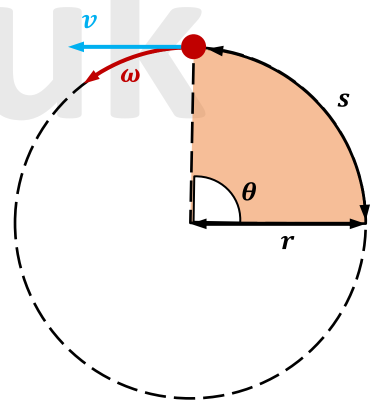
$$v = \omega r$$

Where:

ω = angular velocity measured in $rad\ s^{-1}$.

v = linear speed in ms^{-1}

r = radius of the circle of rotation in m



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Frequency and Period

Circular motion has a frequency and a period.

The frequency, f, is defined as the number of complete revolutions per second ($rev\ s^{-1}$ or $hertz, Hz$).

The period, T, is the time taken for complete revolution (in seconds).

Frequency and period are linked by the equation:

$$f = \frac{1}{T}$$

For a complete circle, an object rotates through 2π radians in a time, T. As a result, the angular speed equation becomes:

$$\omega = \frac{2\pi}{T}$$

Now substituting $f = \frac{1}{T}$ into the equation above, you get an equation that relates ω and f:

$$\omega = 2\pi f$$

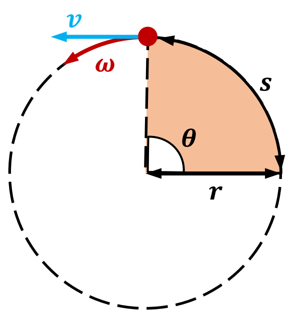
We can substitute the above into $v = \omega r$, to get:

$$v = 2\pi f r$$

Or

$$v = \frac{2\pi r}{T}$$

The above gets us the linear velocity.



Points to note

1) The linear velocity (v) always acts along the tangent to the circle (i.e. at $90°$ to the string).
2) Angular velocity acts along the circular path.
3) Both angular speed and linear speed is constant therefore the magnitude of the velocity remains constant, but the direction of the velocity is constantly changing.
4) Angular velocity and angular frequency both us the Greek letter omega (ω) however the formulas are different.

Angular velocity, $\omega = \frac{\theta}{t}$

Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi f$

Please see '8.1.2 Kinematics of Circular Motion worked examples' pack for exam style questions.

For more revision notes, tutorials and worked examples please visit www.tutorpacks.co.uk.