# AS Level Physics

## Chapter 6 – Newton's Law of Motion and Momentum

## 6.1.1 Newton's Laws of Motion

## Notes

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# NEWTON’S THREE LAWS OF MOTION

## Newton’s 1st Law of Motion

Newton’s 1st Law of motion states:

**“an object will remain at rest (or stationary) or continue to travel with a constant velocity unless acted upon by a resultant force”**

- This means an object will remain still or move with a constant speed, in a straight line, in the same direction unless acted upon by a resultant (net) or unbalanced force.
- If an object has a zero resultant force acting on it, the object is said to be at rest or moves with a constant velocity (or uniform motion).
- Zero resultant force means the combined effect of all the forces acting on the object is zero.
- Therefore FORCE is something that tries to change the state of rest or uniform motion of an object, either through constant or ‘field’ action (i.e. electric, gravity or magnetic fields).

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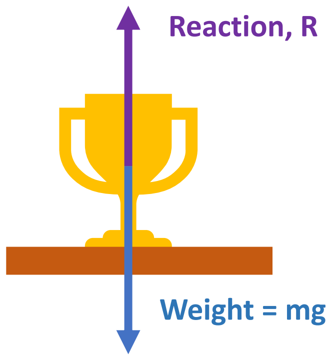
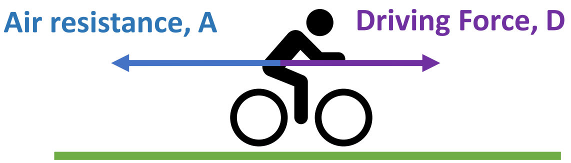
# NEWTON’S THREE LAWS OF MOTION

## Newton’s 1st Law of Motion

Examples:



The trophy is stationary or at rest because the reaction force is equal to the weight of the object. This means the forces are balanced and the resultant force is zero. The trophy will only move when a resultant force acts on it, i.e. if someone comes and pushes the trophy over.



The cyclist is moving at a constant velocity because the driving force from the cyclist is equal to the air resistance. This means the cyclist is moving in the same direction with a constant speed. Therefore the forces are balanced and the resultant force is zero. The cyclist will continue to move in the same direction with the same speed unless a resultant force acts on him i.e. he chooses to speed up or change direction.

### INERTIA

- It is the natural tendency of every object to resist the change in their state of motion. If they are at rest, they want to stay at rest and if they’re moving with a constant velocity they would want to continue to do so. This tendency of objects is called INERTIA.
- Therefore, INERTIA is a resistance to a change in motion (both speed and direction). Unless an external force causes a change, objects want to remain at rest or in motion.
- All objects have inertia and the greater the mass of an object, the greater is its inertia. This results in applying a greater resultant force in order to change the motion of the object.
- For example it is more difficult to change the motion of a box full of bricks compared to changing the motion of an empty box because the box full of bricks weighs a lot more.

# NEWTON'S THREE LAWS OF MOTION

## Newton's 2nd Law of Motion

Newton's 2nd Law of motion states:

**"The rate of change of momentum of an object is directly proportional to the applied resultant force and occurs in the direction of the resultant force"**

- In other words, the resultant force is proportional to the change of momentum per second.
- At GCSE you learn that Newton's 2nd law is defined as $F = ma$ (force = mass x acceleration). At A-level we will look at how this equation is derived from Newton's 2nd law in its general form as stated above. But first you need to know what momentum is:

### Momentum:

The momentum of an object is the product of its mass and velocity.

$$\boldsymbol{p} = \boldsymbol{m} \times \boldsymbol{v}$$

where:

$p$ = Momentum measured in $kg\ ms^{-1}$.
$m$ = mass measured in $kg$.
$v$ = velocity measured in $ms^{-1}$.

Momentum is a vector and therefore has both magnitude and direction. This means momentum to the right can be considered positive and momentum to the left can be negative.

# NEWTON'S THREE LAWS OF MOTION

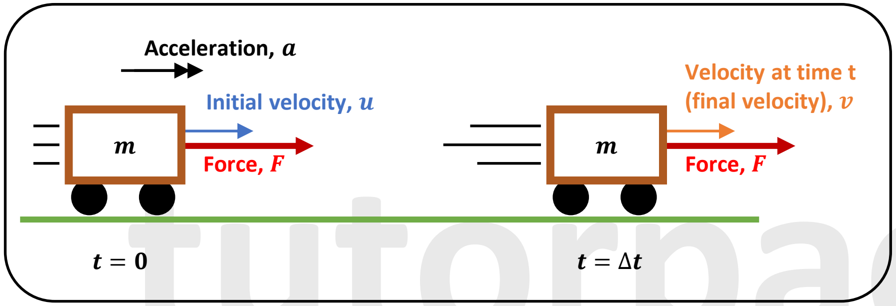
## Newton's 2nd Law of Motion

- Note, force acting on an object can cause a change in its momentum. A large force results in a greater rate at which the objects momentum changes.
- E.g. the harder you throw a ball, the greater is its rate of change of momentum and the more difficult it is to catch.

# NEWTON'S THREE LAWS OF MOTION

## Newton's 2nd Law of Motion

- Now let's look at how $F = ma$ is derived from Newton's 2nd law.
- Consider an object of a constant **mass ($m$)** acted on by a **constant force, F**. This causes the object to move with a **constant acceleration ($a$)** causing the velocity to change from a **initial velocity ($u$),** at **time zero ($t = 0$),** to a **final velocity ($v$)** in a **time ($t = \Delta t$)** without a change of direction.



- The initial momentum of the object:

$$\boldsymbol{p_i = mu}$$

- The final momentum of the object:

$$\boldsymbol{p_f = mv}$$

- Therefore the momentum change:

$$\boldsymbol{\Delta p = final\ momentum - initial\ momentum}$$

$$\boldsymbol{\Delta p = mv - mu}$$

# NEWTON'S THREE LAWS OF MOTION

## Newton's 2nd Law of Motion

- According to Newton's 2nd law, the force is proportional to the rate of change of momentum, therefore:

$$\boldsymbol{F \propto \frac{change\ of\ momentum}{time\ taken} = \frac{\Delta p}{\Delta t}}$$

$$\boldsymbol{F = \frac{mv - mu}{\Delta t}}$$

$$\boldsymbol{F = \frac{m(v-u)}{\Delta t}}$$

$$Therefore, \boldsymbol{F = ma}$$

But we know that acceleration is:

$$a = \frac{(v-u)}{\Delta t}$$

And therefore we substitute $a$ in the equation.

Where $a = \frac{(v-u)}{\Delta t}$ = the acceleration of the object.

- So from Newtons 2nd Law we have confirmed:

$$Resultant\ force\ (F_R) \propto rateof\ change\ of\ momentum\ (= ma)$$

- This proportionality relationship can be written as below:

$$F_R = kma, \qquad where\ k = constant\ of\ proportionality$$

- Typically, k is made to equal 1 by defining the unit force (1 N) as the amount of force that gives an object, of mass 1kg, an acceleration of $1\ ms^{-2}$.
- Therefore, $k = 1$, gives us $F = ma$ following from Newton's 2nd law.
- Note: $F = ma$ is only valid so long as the mass of the object is constant.

# NEWTON'S THREE LAWS OF MOTION

## Worked Example 1:

A football of mass 0.46 kg initially at rest was struck which gave it a velocity of $25ms^{-1}$. The contact time between the foot and the ball was 15ms.

Calculate:

a) The momentum gained by the ball,

b) The average force of impact on the ball.

### *Solution:*

Initially the object is at rest therefore initial velocity, $u = 0ms^{-1}$

a) Using: $change\ of\ momentum = mv - mu$

$$Momentum\ gained = (0.46kg)(25ms^{-1}) - (0.46kg)(0ms^{-1})$$

$$\boldsymbol{Momentum\ gained = 11.5\ kgms^{-1}}$$

**b)** Using: $F = \frac{change\ of\ momentum}{time\ taken} = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{\Delta t}$

$$F = \frac{11.5\ kgms^{-1}}{0.015s} = 766.67\ N$$

$15ms = 15 \times 10^{-3}s = 0.015s$

$$\boldsymbol{Impact\ force = 767\ N}$$

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# NEWTON'S THREE LAWS OF MOTION

## Worked Example 2:

When a ball of mass 0.30 kg hits a wall at a speed of $16\ ms^{-1}$, the ball returns in the direction it came from at a speed of $13\ ms^{-1}$. The contact time was 0.11s. Calculate:

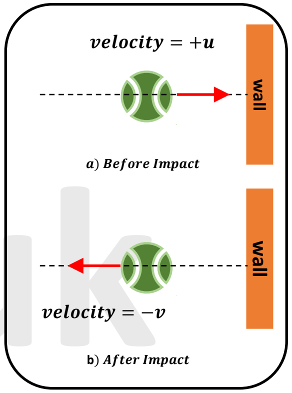
a) The change of momentum of the ball,

b) The impact force on the ball.

### *Solution:*

a) Using: $change\ of\ momentum = mv - mu$

Remember that velocity and momentum are vectors and so they have a magnitude as well as a direction. Suppose the ball hits the wall normally with an initial speed $u$ and it rebounds at speed $v$ in the opposite direction. Since its direction of motion reverses on impact, a sign convention is necessary to represent the two directions. Using + for 'towards the wall' and – for 'away from the wall', its $initial\ momentum = +mu$, and its $final\ velocity = -mv$.



Mass of the ball $m = 0.30kg$, initial velocity $u = +16ms^{-1}$, final velocity $v = -13ms^{-1}$,

$$Change\ of\ momentum = mv - mu$$
$$change\ of\ momentum = (0.30 \times -13) - (0.30 \times 16)$$
$$change\ of\ momentum = -3.9 - 4.8$$
$$\boldsymbol{change\ of\ momentum = -8.7\ kg\ m\ s^{-1}}$$

b) Using: $F = \frac{change\ of\ momentum}{time\ taken} = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{\Delta t}$

$$F = \frac{-8.7\ kgms^{-1}}{0.11s}$$
$$\boldsymbol{F = -79.09N}$$

The minus sign indicates the direction of the force, i.e. to the left

# NEWTON'S THREE LAWS OF MOTION

## Newton's 2nd Law of Motion

### Weight:

To calculate the **WEIGHT (W)** of an object use:

$$\boldsymbol{W = mg}$$

where:
$W$ = weight measured in $Newtons, N$.
$m$ = mass measured in $kg$.
$g$ = acceleration due to gravity measured in N/kg or $ms^{-2}$.

On Earth, acceleration due to gravity is $g = 9.81\ m\ s^{-2}$.

The formula of weight is a version of $F = ma$.

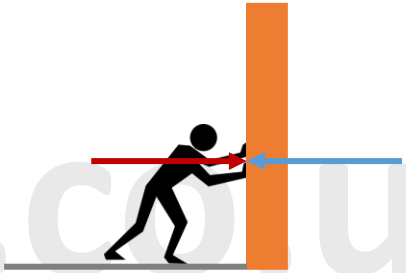
# NEWTON'S THREE LAWS OF MOTION

## Newton's 3rd Law of Motion

Newton's 3rd Law of motion states:

**"If an object A exerts a Force on object B, then object B exerts an equal but opposite force on object A"**

- For example, if you were to push against a wall, then the wall will push back against you, just as hard. But, as soon as you stop pushing, the wall stops too.



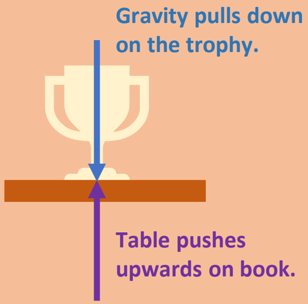
- Newtons 3rd Law can occur in all situations and to all types of forces, however the pairs of forces have to be the same type, e.g. both gravitational or both electrical.

This looks like Newton's 3rd law….

However it's **NOT**

This is because both forces acting on the trophy are not the same type. These are two separate interactions.

Here, the forces are equal and opposite, resulting in zero acceleration, so this is showing Newton's 1st law.

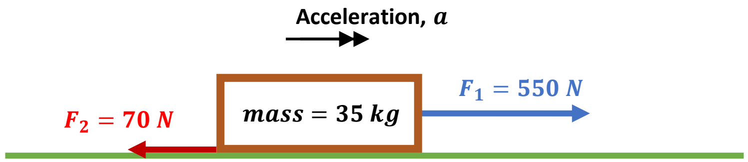


# NEWTONS LAW QUESTIONS

*For more force questions please see Module 3 – Forces and motions pack 3.3 Dynamics*

## Example 1: Resultant Force

A force of 550 N is used to pull a 35 kg box against a constant frictional force of 70 N as shown below. Calculate the acceleration of the box.



From Newton's 2nd law: Resultant force (F) = mass (m) x acceleration (a)

$$F_1 - F_2 = ma$$

$$(550 - 70) = 35a$$

$$a = \frac{480}{35}$$

$$a = \mathbf{13.7\ ms^{-2}\ (1\ d.p.)}$$

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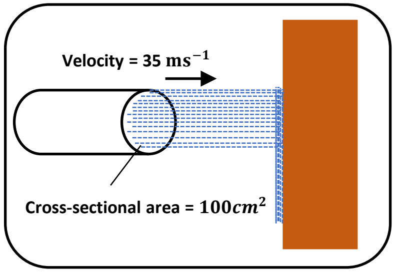
# NEWTONS LAW QUESTIONS

*For more force questions please see Module 3 – Forces and motions pack 3.3 Dynamics*

## Example 2: Hosepipe

A hosepipe with a cross-sectional area of 0.01m² ejects a horizontal jet of water at a speed of $35ms^{-1}$.
The water hits a wall perpendicularly and flows down the wall without rebounding, Calculate the force exerted on the wall. (Density of water = $1 \times 10^3\ kgm^{-3}$).



From Newton's 2nd law:

Force exerted by the wall on the water = Rate of change of momentum of the water

= mass of water striking wall per second x Velocity change of water

= Volume of water/s x Water density x Velocity change of water

$$= (35\ ms^{-1} \times 0.01m^2) \times (1 \times 10^3\ kgm^{-3}) \times (35\ ms^{-1} - 0\ ms^{-1})$$

$$= \underline{\mathbf{1.23 \times 10^4\ N}}$$

From Newton's 3rd law, there must be an equal and opposite force exerted by the water on the wall.

So, force exerted on the wall $= 1.23 \times 10^4 N$

# NEWTONS LAW QUESTIONS

*For more force questions please see Module 3 – Forces and motions pack 3.3 Dynamics*

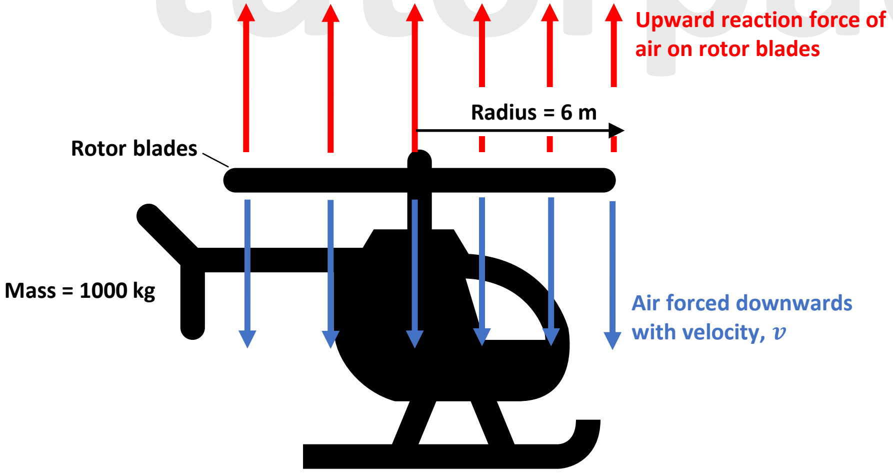
A helicopter can hover in air when the upwards force is equal to the its weight (downwards force). This is possible due to Newton's third law. When helicopter blades rotate they force air downwards and an equal and oppositely directed force is exerted by the air on the blades. This principle can be applied to hovering birds and other hovering bodies/objects.

## Example 3: Helicopter

A hovering helicopter of mass 1000 kg. If the length of each rotor blade is 6 m, calculate the velocity of the air forced downwards by the rotor to keep the helicopter hovering in mid-air.

Density of air = $1.3\ kgm^{-3}$

Acceleration due to gravity, $g = 9.81\ ms^{-2}$



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# NEWTONS LAW QUESTIONS

*For more force questions please see Module 3 – Forces and motions pack 3.3 Dynamics*

## Example 3: Helicopter

From Newton's 2nd law:

Upwards force of air on blades = Rate of change of momentum of air forced down by blades = Helicopter weight

Helicopter weight = mass of air forced down per second x Velocity change of air being forced down

Helicopter weight = Volume of air forced down per second x Density of air x Velocity change of air being forced down

$$mg = \pi R^2 v \times \rho \times v$$

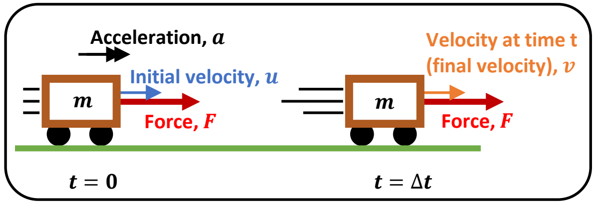
$$v^2 = \frac{mg}{\pi R^2 \rho} = \frac{1000\ kg \times 9.81\ ms^{-2}}{\pi \times 6\ m^2 \times 1.3\ kgm^{-3}}$$

$$v^2 = 66.72 \ldots$$

$$\underline{\boldsymbol{v = 8.17\ ms^{-1}}}$$

# IMPULSE = FΔt

An object with constant mass, $m$, acted upon by a constant force, $F$, which changes its velocity from initial velocity, $u$, to a final velocity, $v$, in time, $t$.



From Newton's 2nd law:

$$Resultant\ force = rate\ of\ change\ of\ momentum$$

$$F = \frac{mv - mu}{\Delta t}$$

Rearranging this equation gives:

$$F\Delta t = mv - mu$$

Therefore:

$$\underset{(Ns\ or\ kgms^{-1})}{Impulse} = \overset{(N)}{F}\underset{(s)}{\Delta t} = \overset{(kg)}{m}\underset{(ms^{-1})}{v} - \overset{(kg)}{m}\underset{(ms^{-1})}{u}$$

$$\underset{(Ns\ or\ kgms^{-1})}{Impulse} = \underset{(N)}{Resultant\ Force} \times \underset{(s)}{time} = \underset{(kgms^{-1})}{momentum\ change}$$

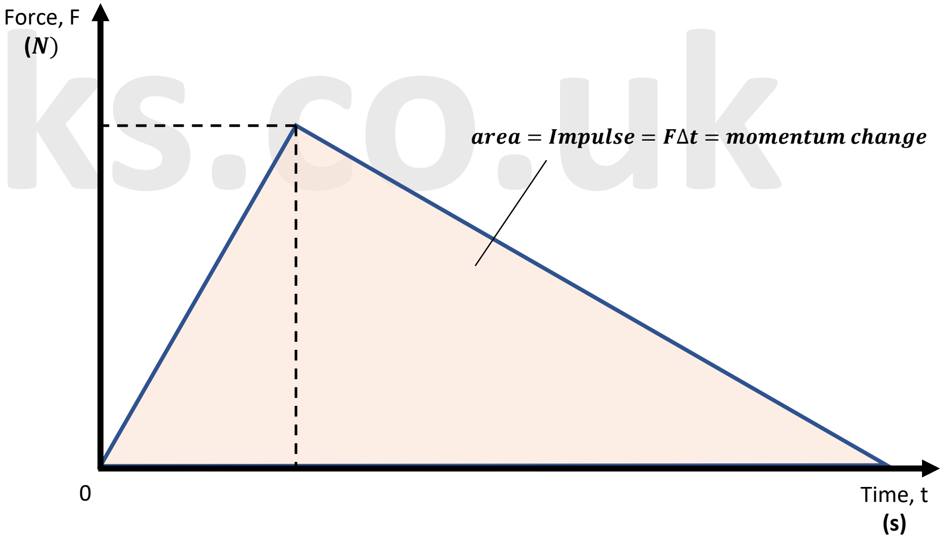
# IMPULSE = FΔt

## Force against time graph

Therefore an Impulse is the product of the magnitude of a force applied on an object and the time during which it is applied.

For example, when you kick a ball, an impulse is applied to the ball. You apply a force on the ball for a short period of time.

The area enclosed by a force against time graph represents the change of momentum and in turn impulse. This shows that the analysis for change in momentum is not only produced by a constant force but can also apply to a varying force.

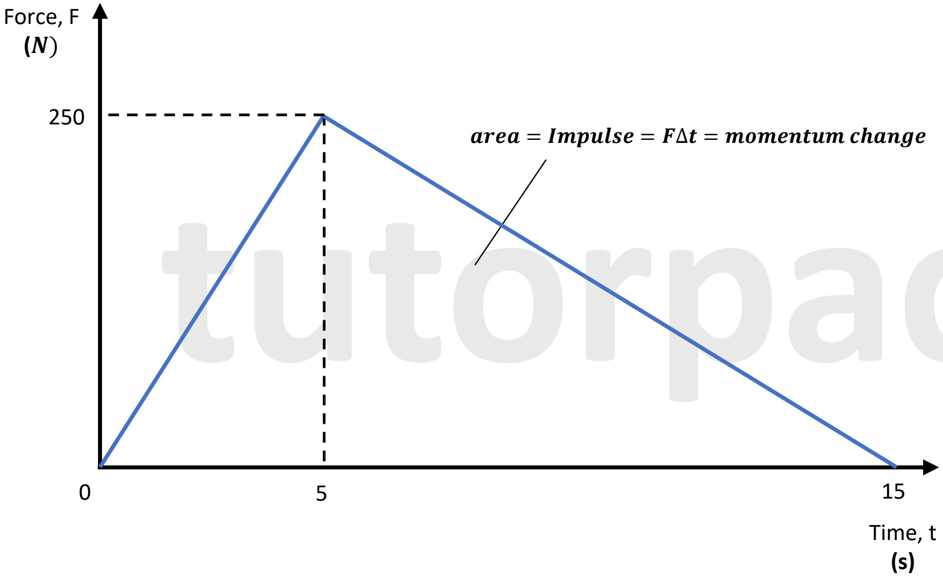


# IMPULSE = FΔt

## Worked Example 1:

A force acting on an object varies with time as demonstrated in the Force-time graph.

a) Calculate the impulse given to the object in a time of 15 seconds using the graph.



***Solution:***

$Impulse = change\ of\ momentum = Area\ enclosed\ by\ the\ F/t\ graph$

$$= \frac{1}{2} \times 15s \times 250N$$

$$= 1875\ Ns\ (or\ kgms^{-1})$$

# IMPULSE = FΔt

## Worked Example 2:

A force of 6 N acts for 12s on a 45kg object which is initially at rest.

Calculate:

a) The change of momentum of the object,

b) The velocity of the object at 12 s.

***Solution:***

a) Using: $\boldsymbol{Impulse = change\ of\ momentum = F\Delta t}$

Therefore:

$$change\ of\ momentum = F\Delta t = 6\ N \times 12\ s$$

$$\boldsymbol{Change\ of\ momentum = 72\ Ns}$$

b) Using: $\boldsymbol{change\ of\ momentum = F\Delta t = mv - mu}$

Initially the object is at rest therefore initial velocity, $u = 0ms^{-1}$

$$72\ Ns = (45\ kg)v - (45kg)(0ms^{-1})$$

$$v = \frac{72Ns}{45kg}$$

$$\boldsymbol{v = 1.6\ ms^{-1}}$$

# IMPULSE = FΔt

**Implications of impulse and change of momentum:**

- Applying a large force for a short time or a small force for a long time gives us the impulse needed to accelerate an object at rest or decelerate a moving object and bring it to rest. This means:

A large duration in time $(\Delta t)$ means a small force $(F)$ exerted for a given change in momentum.

Remember:
$\boldsymbol{F = \frac{mv - mu}{\Delta t}}$,
Therefore if you increase time you reduce the force.

- Crumple zones are used in front of cars to reduce the force exerted on the car and its passengers when involved in a crash. This is done by crumple zones collapsing slowly on impact and increasing the time taken for the car to come to a rest.
- In many sports where you hit a ball (i.e. football, tennis or baseball) players try to increase contact time between the foot, racquet or bat and the ball in order to maximise the balls speed.
- This maximises the impulse given to the ball, producing a greater change in momentum.

Remember:
$\boldsymbol{Impulse = F\Delta t = mv - mu}$
Therefore if you increase contact time and speed the impulse also increases.

# Particle Momentum (Edexcel Only)

We can combine kinetic energy and momentum in order to produce a formula that gives kinetic energy in terms of momentum and mass.

Kinetic energy: $E_k = \frac{1}{2}mv^2$ ……… $(1)$

Momentum: $\rho = mv$

Rearranging gives momentum for $v$ gives us:

$$v = \frac{\rho}{m} \text{………} (2)$$

Now substitute equation (2) into equation (1):

$$E_k = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$

$$E_k = \frac{1}{2}\frac{p^2}{m}$$

$$\therefore E_k = \frac{p^2}{2m}$$

When dealing with kinetic energy of subatomic particles travelling at non-relativistic speed (speeds that are much slower than the speed of light) the above formula is very useful.

**Please see ‘6.1.2 Newton’s Law of Motion Worked Examples’ pack for exam style questions.**

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