AS Level Physics

Chapter 6 – Materials

6.2.2 Springs

Worked Examples

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HOOKE'S LAW

Exam Style Question 1

Atoms in a solid are held in position by electrical forces. These electrical forces can be represented by an imaginary 'interatomic spring' between neighbouring atoms, see Fig. 7.1.

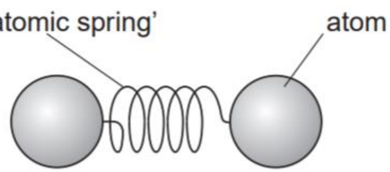


Fig. 7.1

The interatomic spring obeys Hooke's law and has a force constant just as a normal spring in the laboratory. Researchers in America have recently managed to determine the force experienced by an individual atom of cobalt when the atoms are squeezed together. The researchers found that a force of $210\ pN$ changed the separation between a pair of atoms by a distance of $0.16\ nm$.

a) State Hooke's law.

b) Calculate the force constant of the interatomic spring for a pair of cobalt atoms.

HOOKE'S LAW

Exam Style Question 1

Answer:

a) State Hooke's law

Extension is proportional to force applied as long as the elastic limit is not exceeded.

b) Calculate the force constant of the interatomic spring for a pair of cobalt atoms.

Step 1: Convert all units to SI units:

$$210\ pN = 210 \times 10^{-12}\ N$$
$$0.16\ nm = 0.16 \times 10^{-9}\ m$$

Step 2: Use the formula $F = kx$ and rearrange for k:

$$k = \frac{F}{x}$$
$$k = \frac{210 \times 10^{-12}\ N}{0.16 \times 10^{-9}\ m}$$
$$k = 1.3125\ N\ m^{-1}$$

Therefore, the force constant, k is $1.31\ N\ m^{-1}$.

HOOKE'S LAW

Exam Style Question 2

Fig. 6.1 shows a force against extension graph for a spring.

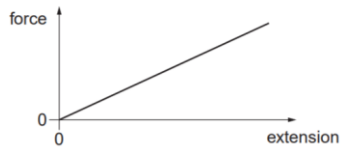


Fig. 6.1

a) Explain how the graph shows that the wire obeys Hooke's law.

b) State what the gradient of the graph represents.

c) Describe how the graph can be used to determine the work done on the spring.

d) Two identical springs are connected end-to-end (series). The force constant of each spring is k. The free ends of the springs are pulled apart as shown in Fig. 6.2.

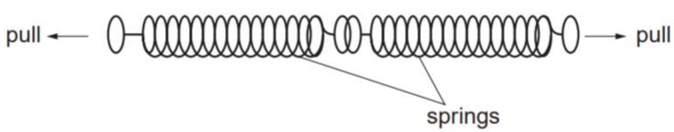


Fig. 6.2

Explain why the force constant of this combination of two springs in series is $\frac{k}{2}$.

HOOKE'S LAW

Exam Style Question 2

Answer:

a) Explain how the graph shows that the wire obeys Hooke's Law.

The graph is a straight line through the origin. This shows that F is proportional to x.

b) State what the gradient of the graph represents.

Force constant (k).

c) Describe how the graph can be used to determine the work done on the spring.

Area under the graph.

d) Explain why the force constant of this combination of two springs in series is $\frac{k}{2}$.

By having two identical springs in series you are essentially doubling the extension.

The force on each spring stays the same.

Using $force\ constant = \frac{force}{extension}$, we know that the force constant is proportional to the force and inversely proportional to the extension. As the force is constant, and the extension is doubled, this means force constant is halved.

ELASTIC POTENTIAL ENERGY

Exam Style Question 3

A light spring of unextended length $2.0\ cm$ is hung from a fixed point. An object of weight $3.0\ N$ is hung from the other end of the spring. Fig. 7.1 shows the length of the spring when the object is in equilibrium.

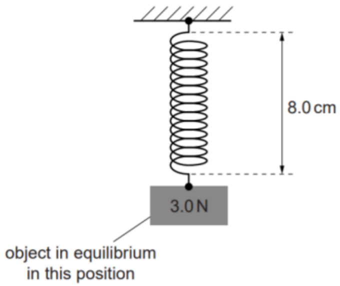


Fig. 7.1

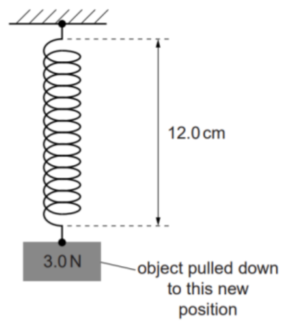


Fig. 7.2

a) Show that the force constant of the spring is $50\ N\ m^{-1}$.

b) The object is pulled vertically downwards. Fig 7.2. shows the new length of the spring.

i) Calculate the change in the elastic potential energy ΔE in the spring.

ii) The object is released from its position shown in Fig. 7.2. Calculate the initial upward acceleration a of the object.

ELASTIC POTENTIAL ENERGY

Exam Style Question 3

Answer:

a) Show that the force constant of the spring is $50\ N\ m^{-1}$.

Use $F = kx$ and rearrange for k:

$$k = \frac{F}{x}$$
$$k = \frac{3.0\ N}{8.0\ cm - 2.0\ cm}$$
$$k = \frac{3.0\ N}{6.0\ cm} = \frac{3.0\ N}{0.06\ m}$$
$$k = 50\ N\ m^{-1}$$

The unextended length of the light spring is 2.0 cm and Fig 7.1 shows that the spring is extended to 8.0 cm. Therefore the extension is $8.0\ cm - 2.0\ cm = 6.0\ cm.$

bi) Calculate the change in the elastic potential energy ΔE in the spring.

Here we are calculating the elastic PE between Fig 7.1 and Fig 7.2. Remember the same spring is used therefore the unextended length stays the same at 2 cm.

Use: $E = \frac{1}{2}kx^2$

$$E_i = \frac{1}{2} \times 50\ N\ m^{-1} \times (0.08\ m - 0.02\ m)^2 = 0.09\ J$$
$$E_f = \frac{1}{2} \times 50\ N\ m^{-1} \times (0.12\ m - 0.02\ m)^2 = 0.25\ J$$
$$\Delta E = E_f - E_i = 0.25\ J - 0.09\ J$$
$$\Delta E = 0.16\ J$$

bii) Calculate the initial upward acceleration a of the object.

Step 1: Use $F = kx$ to calculate the tension in the spring.

$$F = 50\ N\ m^{-1} \times (0.12\ m - 0.02\ m)$$
$$F = 50\ N\ m^{-1} \times 0.10\ m = 5\ N$$

Therefore, the tension in the spring is $5.0\ N$.

Step 2: Calculate the net force acting on the object:

$$F_{net} = 5.0\ N - 3.0\ N = 2.0\ N$$

Step 3: Calculate the mass of the object:

$$m = \frac{W}{g} = \frac{3.0\ N}{9.81\ m\ s^{-2}} = 0.30581 \ldots kg$$

Step 4: Use calculated net force and mass in $F = ma$ and rearrange for a:

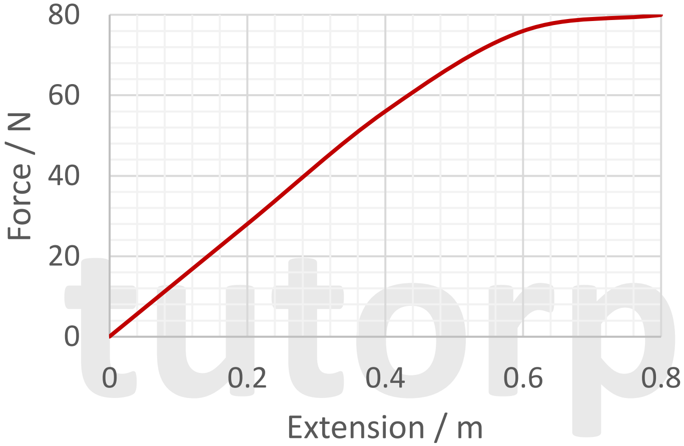
$$a = \frac{F}{m} = \frac{2.0\ N}{0.30581\ kg} = 6.54\ m\ s^{-2}$$

Therefore, the initial upward acceleration of the object is $6.54\ m\ s^{-2}$.

ELASTIC POTENTIAL ENERGY

Exam Style Question 4

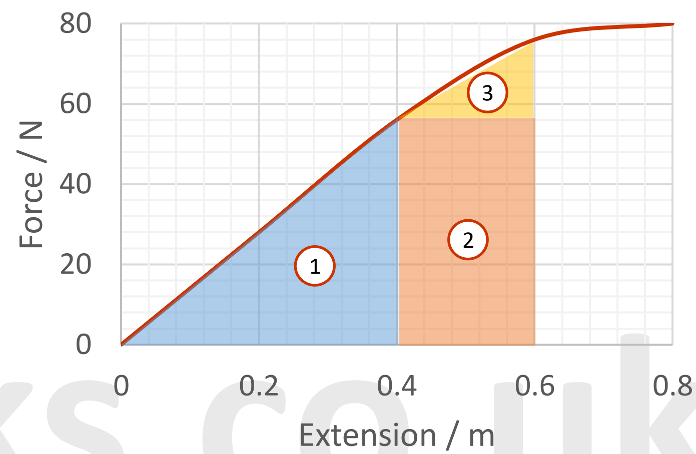
Shown below is a force-extension graph for a metal spring. Find the elastic potential energy stored in the spring when the extension is $0.6\ m$.



ELASTIC POTENTIAL ENERGY

Exam Style Question 4

Answer:
Find the elastic potential energy.



To find the elastic strain energy stored, you need to find the area under the graph shown highlighted below.

You can approximate the area using triangles and rectangles:

Area 1: $\frac{1}{2} \times 56\ N \times 0.4\ m = 11.2\ J$

Area 2: $56\ N \times 0.2\ m = 11.2\ J$

Area 3: $\frac{1}{2} \times 20\ N \times 0.2\ m = 2\ J$

$$Total\ energy \approx 11.2\ J + 11.2\ J + 2\ J = 24.4\ J$$

You can also approximate the area using the squares on the grid:
There are (approximately) 6 big squares, each worth: $20\ N \times 0.2\ m = 4\ J$.
So the total energy stored is $\approx 6 \times 4\ J = 24\ J$.

HOOKE'S LAW

Exam Style Question 5

a) Define the Young modulus of a material and state the condition when it applies.

b) A guitar string has length $0.70\ m$ and cross-sectional area $0.20\ mm^2$. A constant tension of $4.2\ N$ is applied to the string causing a strain of 0.015. Calculate:

i) The stress in the string,

ii) The Young modulus of the material of the string,

iii) The elastic potential energy (stored energy) in the string.

HOOKE'S LAW

Exam Style Question 5

Answer:

a) Define Young modulus and state the condition when it applies.

$Young\ modulus = \frac{stress}{strain}$.

As long as the elastic limit is not exceeded and Hooke's law is obeyed.

bi) The stress in the string.

$$\boldsymbol{stress = \frac{F}{A}}$$

$$stress = \frac{4.2\ N}{0.20\ mm^2} = \frac{4.2\ N}{0.20 \times 10^{-6}\ m^2}$$

$$stress = 2.1 \times 10^7\ Pa$$

bii) The Young modulus of the material of the string.

$$Young\ modulus = \frac{stress}{strain}$$

$$Young\ modulus = \frac{2.1 \times 10^7\ Pa}{0.015}$$

$$Young\ modulus = 1.4 \times 10^9\ Pa$$

biii) The elastic potential energy (stored energy) in the string.

Use $energy = \frac{1}{2}Fx$

Calculate x by rearranging the strain equation:

$$\boldsymbol{strain = \frac{x}{L}}$$

$$x = strain \times L$$

$$x = 0.015 \times 0.70\ m = 0.0105\ m$$

Now sub x back into the energy equation:

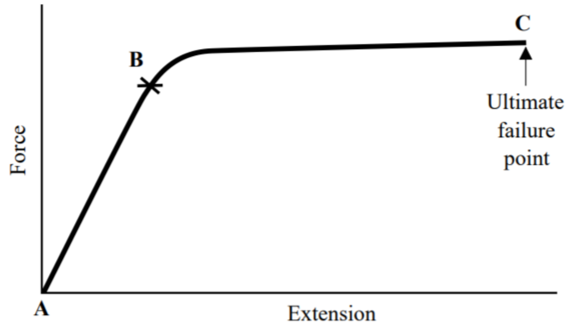
$$energy = \frac{1}{2} \times 4.2\ N \times 0.0105\ m$$

$$\therefore energy = 2.2 \times 10^{-2}\ J$$

HOOKE'S LAW

Exam Style Question 6

The graph shows how a sample of material behaves when extended by a force.



a) What does point **B** represent?

b) State the physical property represented by the gradient of the section **AB** of the graph.

c) What is the ultimate failure point?

d) Explain the significance of the area underneath the line from **A** to **C**.

HOOKE'S LAW

Exam Style Question 6

Answer:

a) What does point B represent?
Hooke's law limit or limit of proportionality.

b) State the physical property represented by the gradient of the section AB of the graph.
Stiffness of sample (force constant).

c) What is the ultimate failure point?
This is the maximum stress that the material can withstand while being stretched or pulled before breaking.

d) Explain the significance of the area underneath the line from A to C.
Work done to stretch the wire to fracture.

Young Modulus

Exam Style Question 7

ai) Describe the behaviour of a wire that obeys Hooke's law.

aii) Define the Young's modulus of a material and state the unit in which it is measured.

b) A student is required to carry out an experiment and draw a suitable graph in order to obtain a value for the Young modulus of a material in the form of a wire. A long, uniform wire is suspended vertically and a weight, sufficient to make the wire taut, is fixed to the free end. The student increases the load gradually by adding known weights. As each weight is added, the extension of the wire is measured accurately.

i) What other quantities must be measured before the value of the Young modulus can be obtained?

ii) Explain how the student may obtain a value of the Young modulus.

iii) How would a value for the elastic energy stored in the wire be found from the results?

Young Modulus

Exam Style Question 7

Answer:

ai) Describe the behaviour of a wire that obeys Hooke's law.
The extension produced by a force in a wire is directly proportional to the force applied.

aii) Define the Young's modulus of a material and state the unit in which it is measured.
The Young modulus is the ratio of stress to strain and measured in $Pa \text{ or } N\,m^{-2}$.

bi) What other quantities must be measured before the value of the Young's modulus can be obtained.

- Length of wire
- Diameter of wire

bii) Explain how the student may obtain a value of the Young's modulus.
From the gradient of the stress vs. strain graph.
Where $stress = \frac{F}{A}$ and $strain = \frac{x}{l}$.

biii) How would a value for the elastic energy stored in the wire be found from the results?
Area under the line of the Force vs. extension graph.

Young Modulus

Exam Style Question 8

Fig. 8.1 shows a graph of stress against strain for rubber.

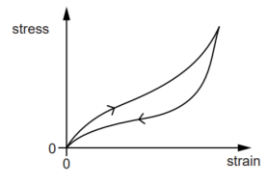


Fig. 8.1

a) Use Fig. 8.1 to describe the main physical properties of this material.

b) Fig 8.2 shows a metal strip pulled from its ends until it breaks.

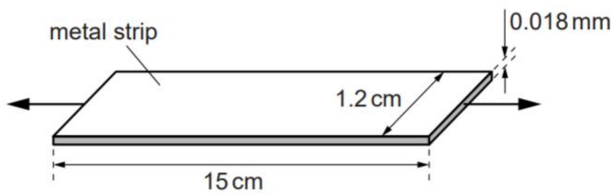


Fig. 8.2

The strip is $15\ cm$ long, $1.2\ cm$ wide and $0.018mm$ thick. The breaking force for this strip is $16\ N$. The Young modulus of the metal is $7.1 \times 10^{10}\ Pa$.

i) Calculate the extension of the metal strip when it breaks. State one assumption made in your calculation.

ii) Calculate the breaking force of a rod of radius $0.60\ cm$ made from the same metal.

Young Modulus

Exam Style Question 8

Answer:

a) Use Fig. 8.1 to describe the main physical properties of this material.

The material is elastic. The material returns to its original shape when force is removed and there is no plastic deformation.

Strain is zero when stress is removed.

It does not obey Hooke's law.

The loading and unloading graphs are different.

bi) Calculate the extension of the metal strip when its breaks. State one assumption made in your calculation.

Calculate stress:

$$\therefore stress = \frac{F}{A} = \frac{16\ N}{(1.2 \times 10^{-2}\ m)(0.018 \times 10^{-3}\ m)}$$

$$stress = 7.41 \times 10^{7}\ Pa$$

Now use Young's modulus and rearrange for strain:

$$Young\ modulus = \frac{stress}{strain}$$

$$\therefore strain = \frac{stress}{Young\ modulus} = \frac{7.41 \ldots \times 10^{7}\ Pa}{7.1 \times 10^{10}\ Pa}$$

$$Strain = 1.043 \ldots \times 10^{-3}$$

Using $strain = \frac{x}{l}$ we can calculate the extension:

$$x = strain \times l = 1.043 \times 10^{-3} \times 0.15\ m$$

$$x = 1.564 \ldots \times 10^{-4}\ m$$

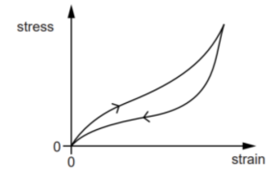
Therefore, the extension is $1.6 \times 10^{-4}\ m$.

Assumption: Hooke's law is obeyed and elastic limit is not exceeded.

Young Modulus

Exam Style Question 8

Fig. 8.1 shows a graph of stress against strain for rubber.



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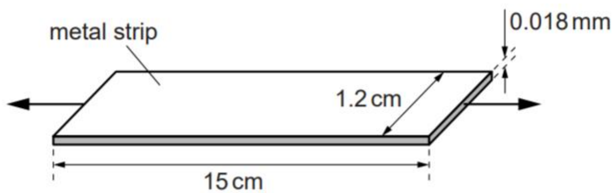

Fig. 8.2

The strip is $15\ cm$ long, $1.2\ cm$ wide and $0.018 mm$ thick. The breaking force for this strip is $16\ N$. The Young modulus of the metal is $7.1 \times 10^{10}\ Pa$.

i) Calculate the extension of the metal strip when its breaks. State one assumption made in your calculation.

ii) Calculate the breaking force of a rod of radius $0.60\ cm$ made from the same metal.

Young Modulus

Exam Style Question 8

Answer:

bii) Calculate the breaking force of a rod of radius $0.60\ cm$ made from the same metal.

As the rod is made from the same material as the metal strip the stress is the same $(7.41 \times 10^7\ Pa)$.

Use $stress = \frac{F}{A}$ and rearrange for F:

$$F = stress \times A$$
$$F = (7.41 \times 10^7\ Pa) \times (\pi(0.60 \times 10^{-2})^2)$$
$$F = 8380.512563\ N$$

Therefore, the breaking force is $8.4 \times 10^3\ N$.

Remember the area of the circle is πr^2.

Young Modulus

Exam Style Question 9

a) Fig. 6.1 shows the stress against strain graphs of two materials X and Y. Describe the properties of materials X and Y.

Fig. 6.1

b) You are given a spring, a metre rule and a $100\ g$ mass. Describe how you would determine the force constant k of the spring.

c) A glider of mass $0.180\ kg$ is placed on a horizontal frictionless air track. One end of the glider is attached to a compressible spring of force constant $50\ N\ m^{-1}$. The glider is pushed against a fixed support so that the spring compresses by $0.070\ m$, see Fig. 6.2. The glider is then released.

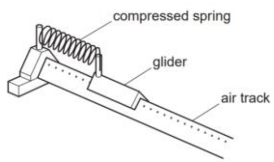

Fig. 6.2

i) Calculate the horizontal acceleration of the glider immediately after release.

ii) After release, the spring exerts a force on the glider for a time of $0.094\ s$. Calculate the average rate of work done by the spring on the glider.

Young Modulus

Exam Style Question 9

Answer:

a) Describe the properties of materials X and Y.

Material X: It is a brittle material. It returns to the same length when stress/force is removed. X obeys Hooke's law.

Material Y: It is a polymeric material. It is elastic and it returns to same length when stress/force is removed. Y does not obey Hooke's law.

b) Describe how you would determine the force constant k of the spring.

Step 1: Use the ruler to determine the original length of the spring.
Step 2: Hang $100\ g$ mass from the spring.
Step 3: Determine the extension (x) using the ruler.
Step 4: Convert $100\ g$ into newtons using

$$W = mg = (0.1\ kg)(9.81\ m\ s^{-2}) = 0.981\ N$$

Step 5: Use $F = \frac{k}{x}$ and rearrange for k:

$$k = \frac{F}{x}$$

$$k = \frac{0.981\ N}{x}$$

ci) Calculate the horizontal acceleration of the glider immediately after release.

Use $F = kx$

$$F = 50\ N\ m^{-1} \times 0.070\ m$$

$$F = 3.5\ N$$

Now use $F = ma$ and rearrange for a:

$$a = \frac{F}{m} = \frac{3.5\ N}{0.180\ kg} = 19.4444\ldots\ \mathrm{m\ s^{-2}}$$

So, the horizontal acceleration of the glider is $19.4\ m\ s^{-2}$.

Young Modulus

Exam Style Question 9

a) Fig. 6.1 shows the stress against strain graphs of two materials X and Y. Describe the properties of materials X and Y.

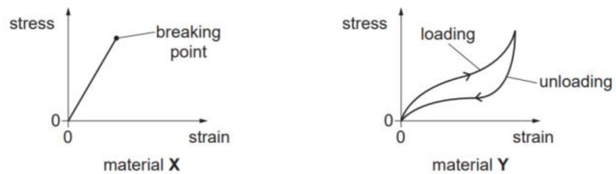

Fig. 6.1

b) You are given a spring, a metre rule and a $100\ g$ mass. Describe how you would determine the force constant k of the spring.

c) A glider of mass $0.180\ kg$ is placed on a horizontal frictionless air track. One end of the glider is attached to a compressible spring of force constant $50\ N\ m^{-1}$. The glider is pushed against a fixed support so that the spring compresses by $0.070\ m$, see Fig. 6.2. The glider is then released.

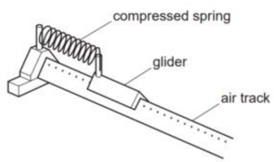

Fig. 6.2

i) Calculate the horizontal acceleration of the glider immediately after release.

ii) After release, the spring exerts a force on the glider for a time of $0.094\ s$. Calculate the average rate of work done by the spring on the glider.

Young Modulus

Exam Style Question 9

Answer:

cii) Calculate the average rate of work done by the spring on the glider.

First calculate the average work done

$$average\ work\ done = average\ force \times displacement$$

$$average\ work\ done = \frac{3.5\ N}{2} \times 0.070\ m$$

$$average\ work\ done = 1.75\ N \times 0.070\ m$$

$$average\ work\ done = 0.1225\ J$$

Now calculate the average rate of work done:

$$average\ rate\ of\ work\ done = \frac{average\ work\ done}{time}$$

$$average\ rate\ of\ work\ done = \frac{0.1225\ J}{0.094\ s}$$

$$average\ rate\ of\ work\ done = 1.303 \ldots J\ s^{-1}$$

Therefore, the average rate of work done by the spring on the glider $1.3\ J\ s^{-1}$.

To calculate the average force just divide it by two. This is because the force goes from 0 N to 3.5 N.

Please see **'6.2.1 Springs notes'** pack for revision notes.

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