A2 Level Physics

Chapter 9 – Thermal Physics

9.4.2 Ideal Gases

Worked Examples

Ideal Gases

Exam Style Question 1

(a) (i) State, in words, Boyle's law.

(ii) Fig. 6.1 is a graph showing the relationship between the quantities involved in Boyle's law. Label the axes appropriately.

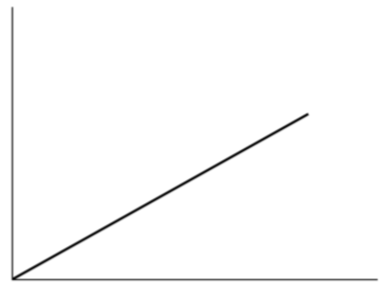

Fig. 6.1

(b) A gas cylinder of internal volume $0.050\ m^3$ contains compressed air at $21\ °C$ and pressure $1.2\ \times\ 10^7\ Pa$. The molar mass of air is $0.029\ kg\ mol^{-1}$.

(i) Calculate

1) the number of moles of air in the cylinder

2) the mass of air in the cylinder.

(ii) An additional $1.5\ m^3$ of air at $21\ °C$ and at atmospheric pressure, $1.0 \times 10^5\ Pa$, is pumped into the cylinder. Calculate the new pressure of air in the cylinder, assuming no change in temperature during the process.

Ideal Gases

Exam Style Question 1

(a)(i) State, in words, Boyle's law.

For a fixed mass of gas at constant temperature pressure is inversely proportional to volume.

(ii) Fig. 6.1 is a graph showing the relationship between the quantities involved in Boyle's law. Label the axes appropriately.

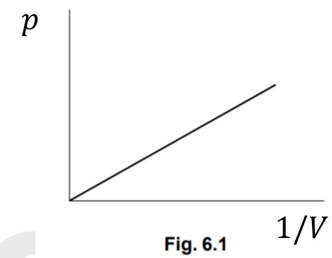

Fig. 6.1

(b) (i) 1) Calculate the number of moles of air in the cylinder.

Use $pV = nRT$ and rearrange for n

$$n = \frac{pV}{RT} = \frac{(1.2 \times 10^7\ Pa)(0.05\ m^3)}{(8.31 JK^{-1} mol^{-1})(21℃ + 273)}$$

$$n = 246$$

(b) (i) (2) Calculate the mass of air in the cylinder.

Use $n = \frac{m}{M}$ and rearrange for m

$$m = nM = (246)(0.029\ kg\ mol^{-1})$$

$$m = 7.1\ kg$$

Ideal Gases

Exam Style Question 1

(a) (i) State, in words, Boyle's law.

(ii) Fig. 6.1 is a graph showing the relationship between the quantities involved in Boyle's law. Label the axes appropriately.

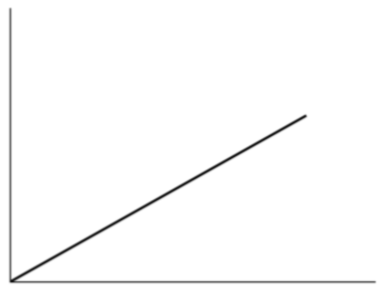

Fig. 6.1

(b) A gas cylinder of internal volume $0.050\ m^3$ contains compressed air at $21\ °C$ and pressure $1.2\ \times\ 10^7\ Pa$. The molar mass of air is $0.029\ kg\ mol^{-1}$.

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(ii) An additional $1.5\ m^3$ of air at $21\ °C$ and at atmospheric pressure, $1.0 \times 10^5\ Pa$, is pumped into the cylinder. Calculate the new pressure of air in the cylinder, assuming no change in temperature during the process.

Ideal Gases

Exam Style Question 1

(b) (ii) Calculate the new pressure of air in the cylinder, assuming no change in temperature during the process.

Step 1: Calculate the new number of moles that have been introduced into the cylinder:

$$n_{air\ added} = \frac{pV}{RT} = \frac{(1.0 \times 10^5\ Pa)(1.5\ m^3)}{(8.31 J K^{-1} mol^{-1})(21℃ + 273)}$$

$$n_{air\ added} = 61.4$$

Step 2: Find out the total number of moles present in the cylinder:

$$n_{total} = n_{initial} + n_{air\ added} = 246 + 61.4$$

$$n_{total} = 307.4$$

Step 3: Calculate the new final pressure using:

$$pV = nRT$$

$$p_{final} = \frac{nRT}{V} = \frac{(307.4)(8.31\ J\ K^{-1}\ mol^{-1})(21℃ + 273)}{0.050\ m^3}$$

$$p_{final} = 1.5 \times 10^7\ Pa$$

Ideal Gases

Exam Style Question 2

(a) State a conclusion about the movement of gas molecules provided by observations of Brownian motion.

(b) Fig. 5.1 shows a gas contained in a cylinder enclosed by a piston. The volume of the gas inside the cylinder is 120 cm^3. The pressure inside the cylinder is 350 kPa.

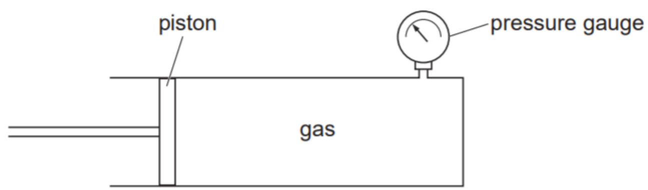

Fig. 5.1

(i) State a necessary condition for Boyle's law to apply to a fixed quantity of gas.

(ii) The piston in Fig. 5.1 is moved quickly so that the gas occupies a volume of 55 cm^3. Use Boyle's law to calculate the new pressure of the gas.

(iii) In practice, the quick movement of the piston during compression of the gas causes an increase in the temperature of the gas. Explain this increase in temperature in terms of the movement of the piston and the motion of the gas molecules.

Ideal Gases

Exam Style Question 2

(a)State a conclusion about the movement of gas molecules provided by observations of Brownian motion.

Gas molecules move in random haphazard motion.

(b) Fig. 5.1 shows a gas contained in a cylinder enclosed by a piston. The volume of the gas inside the cylinder is $120\ cm^3$. The pressure inside the cylinder is $350\ kPa$.

(i) State a necessary condition for Boyle's law to apply to a fixed quantity of gas.

Constant temperature.

(ii) The piston in Fig. 5.1 is moved quickly so that the gas occupies a volume of $55\ cm^3$. Use Boyle's law to calculate the new pressure of the gas.

Use $P_1V_1 = P_2V_2$

$$V_1 = 55\ cm^3 = 120 \times 10^{-4}\ m^3$$
$$V_2 = 55\ cm^2 = 55 \times 10^{-4}\ m^3$$
$$(350000\ Pa)(120 \times 10^{-4}\ m^3) = P_2 \times (55 \times 10^{-4}\ m^3)$$
$$P_2 = \frac{(350000\ Pa)(120 \times 10^{-4}\ m^3)}{(55 \times 10^{-4}\ m^3)}$$
$$P_2 = 763636\ Pa = 764\ kPa$$

(iii) Explain this increase in temperature in terms of the movement of the piston and the motion of the gas molecules.

When a molecule collides with the moving piston it rebounds with higher speed and higher KE. And as $KE = \frac{3}{2}kT$, KE is proportional to temperature and therefore as the KE increases so does the temperature.

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Ideal Gases

Exam Style Question 3

(a) One assumption required for the development of the kinetic model of a gas is that molecules undergo perfectly elastic collisions with the walls of their containing vessel and with each other.

(i) Explain what is meant by a perfectly elastic collision.

(ii) State three other assumptions of the kinetic theory of gases.

(b) Fig. 5.1 shows a cubical box of side length $0.20\ m$. The box contains one molecule of mass $4.8 \times 10^{-26}\ kg$ moving with a constant speed of $500\ m\ s^{-1}$. The molecule collides elastically at right angles with the opposite faces X and Y of the box.

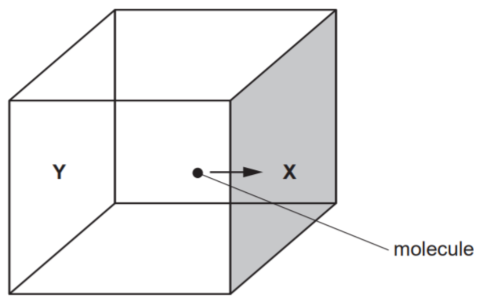

Fig. 5.1

(i) Calculate the change of momentum each time the molecule collides with face X.

(ii) Calculate the number of collisions made by the molecule with face X in $1.0\ s$.

(iii) Calculate the mean force exerted on the molecule by face X.

(iv) Hence state the force exerted on face X by the molecule. Justify your answer.

(c) The single molecule in the box in (b) is replaced by 3 moles of air at atmospheric pressure.
(i) Calculate the number of air molecules in the box.

(ii) Suggest why the pressure exerted by the air on each of the six faces of the box is the same.

(iii) The temperature of the air inside the box is increased. Explain in terms of the motion of the air molecules how the pressure exerted by the air will change.

Ideal Gases

Exam Style Question 3

(a) (i) Explain what is meant by a perfectly elastic collision.
A collision with no loss of KE.

(a) (ii) State three other assumptions of the kinetic theory of gases.
Volume of particles is negligible compared to the volume of the vessel.
No intermolecular forces acting other than during collisions.
Gas consists of a large number of molecules moving randomly.

(b) (i) Calculate the change of momentum each time the molecule collides with face X.
Use $\Delta\rho = mv - mu = m(v-u)$

$$\Delta\rho = (4.8 \times 10^{-26}\ kg)[500\ ms^{-1} - (-500\ ms^{-1})]$$
$$\Delta\rho = 4.8 \times 10^{-23}\ kg\ m\ s^{-1}$$

(b) (ii) Calculate the number of collisions made by the molecule with face X in $1.0\ s$.

Step 1: Calculate the time between collisions using $s = \frac{d}{t}$ and rearrange for t:

$$t = \frac{d}{s} = \frac{0.4\ m}{500\ m\ s^{-1}} = 8 \times 10^{-4}\ s$$

0.4 m because the particle has to travel 0.4m to hit side X again.

Step 2: Now we know how long one collision takes we have to find out how many collisions occur in $1\ s$:

$$Number\ of\ collisions = \frac{1\ s}{8 \times 10^{-4}\ s} = 1250$$

(b) (iii) Calculate the mean force exerted on the molecule by face X.
Remember Force = rate of change of momentum:

$$F = \frac{\Delta\rho}{t} = \frac{(1250)(4.8 \times 10^{-23}\ kg\ m\ s^{-1})}{1\ s}$$
$$F = 6.0 \times 10^{-20}\ N$$

Ideal Gases

Exam Style Question 3

(a) One assumption required for the development of the kinetic model of a gas is that molecules undergo perfectly elastic collisions with the walls of their containing vessel and with each other.

(i) Explain what is meant by a perfectly elastic collision.

(ii) State three other assumptions of the kinetic theory of gases.

(b) Fig. 5.1 shows a cubical box of side length $0.20\ m$. The box contains one molecule of mass $4.8 \times 10^{-26}\ kg$ moving with a constant speed of $500\ m\ s^{-1}$. The molecule collides elastically at right angles with the opposite faces X and Y of the box.

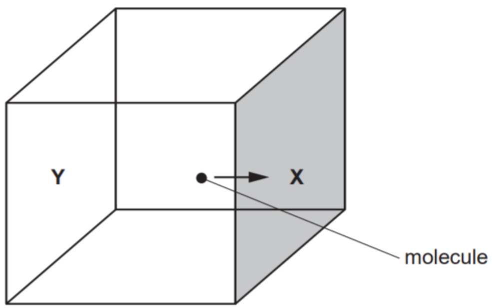

Fig. 5.1

(i) Calculate the change of momentum each time the molecule collides with face X.

(ii) Calculate the number of collisions made by the molecule with face X in $1.0\ s$.

(iii) Calculate the mean force exerted on the molecule by face X.

(iv) Hence state the force exerted on face X by the molecule. Justify your answer.

(c) The single molecule in the box in (b) is replaced by 3 moles of air at atmospheric pressure.
(i) Calculate the number of air molecules in the box.

(ii) Suggest why the pressure exerted by the air on each of the six faces of the box is the same.

(iii) The temperature of the air inside the box is increased. Explain in terms of the motion of the air molecules how the pressure exerted by the air will change.

Ideal Gases

Exam Style Question 3

(b) (iv) Hence state the force exerted on face X by the molecule. Justify your answer.

$6.0 \times 10^{-20}\ N$ due to Newton's third law.

(c) (i) Calculate the number of air molecules in the box.

Use $N = nN_A$

$$N = 3 \times 6 \times 10^{23} = 1.8 \times 10^{24}$$

(c) (ii) Suggest why the pressure exerted by the air on each of the six faces of the box is the same.

Very large number of particles that are moving randomly means that at any instant the number of collisions on each face will be the same.

(c) (iii) The temperature of the air inside the box is increased. Explain in terms of the motion of the air molecules how the pressure exerted by the air will change.

Mean KE of the molecules increases and therefore increased rate of collisions with the wall.

Ideal Gases

Exam Style Question 4

(a) (i) A container has **1 mole** of an ideal gas. The volume of the container is V cubic metres (m^3) and the gas exerts pressure p pascal (Pa). On Fig. 6.1, show the relationship between the product pV and the absolute temperature T of the gas.

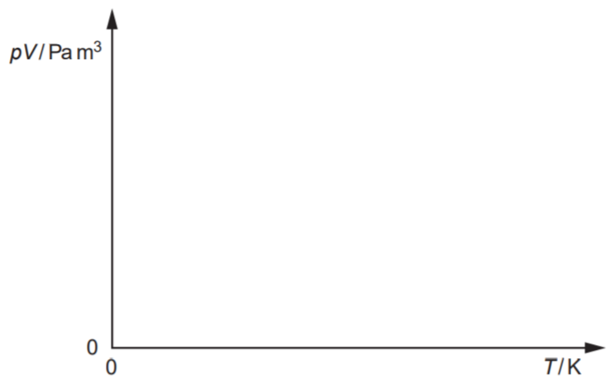

Fig. 6.1

(ii) State the value of the gradient of this graph

(b) The volume of 1.5 moles of an ideal gas at $-40\ °C$ is $2.4 \times 10^{-2}\ m^3$. The gas is now heated at constant pressure p. Calculate

(i) the new volume of the gas at a temperature of 250 °C

(ii) (ii) the value of the pressure p

Ideal Gases

Exam Style Question 4

(a)(i) On Fig. 6.1, show the relationship between the product pV and the absolute temperature T of the gas.

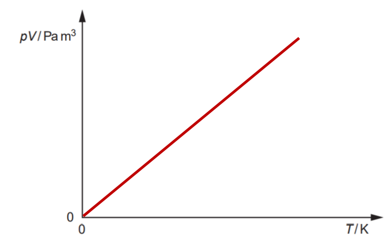

Fig. 6.1

(a) (ii) State the value of the gradient of this graph.

$$8.31\ J\ mol^{-1}\ K^{-1}$$

(b) Calculate

(i) the new volume of the gas at a temperature of 250 °C.

Step 1: Convert degree Celsius (°C) to Kelvins (K)

$$-40\ °C = 233\ K$$
$$250\ °C = 523\ K$$

Step 2: Use $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\frac{2.4 \times 10^{-2}\ m^3}{233\ K} = \frac{V_2}{523\ K}$$
$$V_2 = \frac{2.4 \times 10^{-2}\ m^3}{233\ K} \times 523\ K$$
$$V_2 = 0.054\ m^3$$

(ii) the value of the pressure p

Use $pV = nRT$ and rearrange for p:

$$p = \frac{nRT}{V} = \frac{1.5\ moles \times 8.31\ J\ mol^{-1}K^{-1} \times 233\ K}{2.4 \times 10^{-2}\ m^3}$$
$$p = 1.21 \times 10^5\ Pa$$

Ideal Gases

Exam Style Question 5

(a) The molar mass of hydrogen gas is $2.02 \times 10^{-3}\ kg\ mol^{-1}$. Calculate the mass of a hydrogen molecule.

(b) The temperature of the Earth's upper atmosphere is about $1100\ K$. Show that at this temperature the mean kinetic energy of an air molecule is about $2 \times 10^{-20}\ J$.

(c) Show that the speed of a helium atom of mass $6.6 \times 10^{-27}\ kg$ at a temperature of $1100\ K$ is about $2.5\ km\ s^{-1}$.

(d) The escape velocity from the Earth is $11\ km\ s^{-1}$. The escape velocity is the minimum vertical velocity a particle must have in order to escape from the Earth's gravitational field. Explain why helium atoms still escape from the Earth's atmosphere.

Ideal Gases

Exam Style Question 5

(a)The molar mass of hydrogen gas is $2.02 \times 10^{-3}\ kg\ mol^{-1}$. Calculate the mass of a hydrogen molecule.

(b)

Use $N = \frac{m}{M} N_A$ and rearrange for m:

$$m = \frac{NM}{N_A} = \frac{(1)(2.02 \times 10^{-3}\ kg\ mol^{-1})}{6.02 \times 10^{23}\ mol^{-1}}$$

$$m = 3.4 \times 10^{-27}\ kg$$

(b) Show that at this temperature the mean kinetic energy of an air molecule is about $2 \times 10^{-20}\ J$.

Use $mean\ KE = \frac{3}{2} kT$

$$mean\ KE = \frac{3}{2}(1.38 \times 10^{-23} J\ K^{-1})(1100\ K)$$

$$mean\ KE = 2.3 \times 10^{-20}\ J$$

(c) Show that the speed of a helium atom of mass $6.6 \times 10^{-27}\ kg$ at a temperature of $1100\ K$ is about $2.5\ km\ s^{-1}$.

Use $KE = \frac{1}{2} m v^2$ and rearrange for v

$$v = \sqrt{\frac{KE}{\left(\frac{1}{2}\right)(m)}} = \sqrt{\frac{(2.3 \times 10^{-20}\ J)}{\left(\frac{1}{2}\right)(6.6 \times 10^{-27}\ kg)}}$$

$$v = 2.6 \times 10^{3}\ m\ s^{-1}$$

(d) Explain why helium atoms still escape from the Earth's atmosphere.

What we just calculated in (c) is the mean velocity of the helium atoms but in reality, helium atoms have a range of speeds and hence some atoms have a velocity greater than the escape velocity of $11\ km\ s^{-1}$.

Ideal Gases

Exam Style Question 6

(a) The ideal gas equation may be written as

$$pV = nRT$$

State the meaning of the terms n and T.

(b) Fig. 6.1 shows a cylinder that contains a fixed amount of an ideal gas. The cylinder is fitted with a piston that moves freely. The gas is at a temperature of 20 °C and the initial volume is $1.2 \times 10^{-4}\ m^3$. Fig. 6.2 shows the cylinder after the gas has been heated to a temperature of 90 °C under constant pressure.

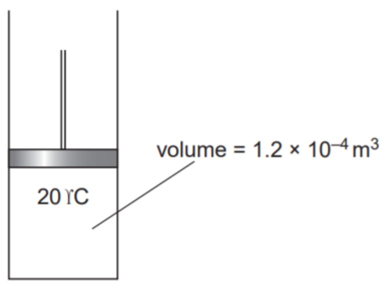

Fig. 6.1

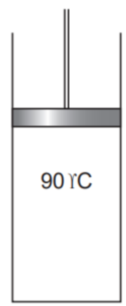

Fig. 6.2

(i) Explain in terms of the motion of the molecules of the gas why the volume of the gas must increase if the pressure is to remain constant as the gas is heated.

(ii) Calculate the volume of the gas at 90 °C.

(c) The mass of each gas molecule is $4.7 \times 10^{-26}\ kg$. Estimate the average speed of the gas molecules at 90 °C.

Ideal Gases

Exam Style Question 6

(a) State the meaning of the terms n and T.

n = number of moles
T = absolute temperature

(b) (i) Explain in terms of the motion of the molecules of the gas why the volume of the gas must increase if the pressure is to remain constant as the gas is heated.

When gas is heater molecules gain KE and move faster. This would cause more collisions per second with the walls. These collisions exert more force and therefore there is a greater change in momentum per collision so pressure would increase. For a constant pressure fewer collisions per second are required and so constant pressure is achieved by increasing the volume. The increase in volume would mean there are fewer collisions per second.

(b) (ii) Calculate the volume of the gas at 90 °C.

Use $\frac{V}{T} = constant$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{1.2 \times 10^{-4}\ m^3}{20℃ + 273} = \frac{V_2}{90℃ + 273}$$

$$V_2 = \left(\frac{1.2 \times 10^{-4}\ m^3}{20℃ + 273}\right)(90℃ + 273)$$

$$V_2 = 1.49 \times 10^{-4}\ m^3$$

Ideal Gases

Exam Style Question 6

(a) The ideal gas equation may be written as

$$pV = nRT$$

State the meaning of the terms n and T.

(b) Fig. 6.1 shows a cylinder that contains a fixed amount of an ideal gas. The cylinder is fitted with a piston that moves freely. The gas is at a temperature of 20 °C and the initial volume is $1.2 \times 10^{-4}\ m^3$. Fig. 6.2 shows the cylinder after the gas has been heated to a temperature of 90 °C under constant pressure.

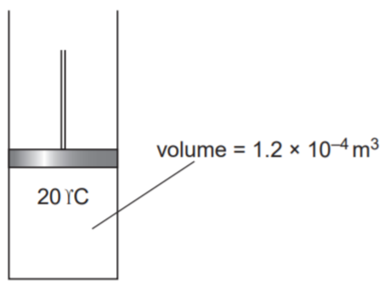

Fig. 6.1

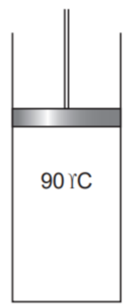

Fig. 6.2

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(ii) Calculate the volume of the gas at 90 °C.

(c) The mass of each gas molecule is $4.7 \times 10^{-26}\ kg$. Estimate the average speed of the gas molecules at 90 °C.

Ideal Gases

Exam Style Question 6

(c) The mass of each gas molecule is $4.7 \times 10^{-26}\ kg$. Estimate the average speed of the gas molecules at 90 °C.

Use $\frac{1}{2}mv^2 = \frac{3}{2}kT$ and rearrange for v

$$v = \sqrt{\frac{\frac{3}{2}kT}{\frac{1}{2}m}}$$

$$v = \sqrt{\frac{\left(\frac{3}{2}\right)(1.38 \times 10^{-23}\ J\ K^{-1})(90℃ + 273)}{\left(\frac{1}{2}\right)(4.7 \times 10^{-26}\ kg)}}$$

$$v = 565\ ms^{-1}$$

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Ideal Gases

Exam Style Question 7

(a) (i) Write down the equation of state for n moles of an ideal gas.

(ii) The molecular kinetic theory leads to the derivation of the equation

$$pV = \frac{1}{3}Nm\overline{c^2},$$

where the symbols have their usual meaning.

State three assumptions that are made in this derivation.

(b) Calculate the average kinetic energy of a gas molecule of an ideal gas at a temperature of $20\ °C$.

(c) Two different gases at the same temperature have molecules with different mean square speeds. Explain why this is possible.

Ideal Gases

Exam Style Question 7

(a)(i) Write down the equation of state for n moles of an ideal gas.

$$pV = nRT$$

(ii) State three assumptions that are made in this derivation.

All particles are identical or have the same mass.
Collisions of gas molecules are elastic.
Inter molecular forces are negligible except during collisions.
Volume of molecules is negligible compared to the volume of the container.
Time of collisions is negligible.
Motion of molecules is random.
Large number of molecules present.

(b) Calculate the average kinetic energy of a gas molecule of an ideal gas at a temperature of $20\ °C$.

Use $KE = \frac{3}{2}kT$

$$KE = \frac{3}{2}(1.38 \times 10^{-23}\ J\ K^{-1})(20°\text{C} + 273)$$
$$KE = 6.1 \times 10^{-21}\ J$$

(c) Two different gases at the same temperature have molecules with different mean square speeds. Explain why this is possible.

Masses are different hence because KE is the same therefore the mean square speeds must be different.

Please see **'9.4.1 Ideal Gases notes'** pack for revision notes.

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